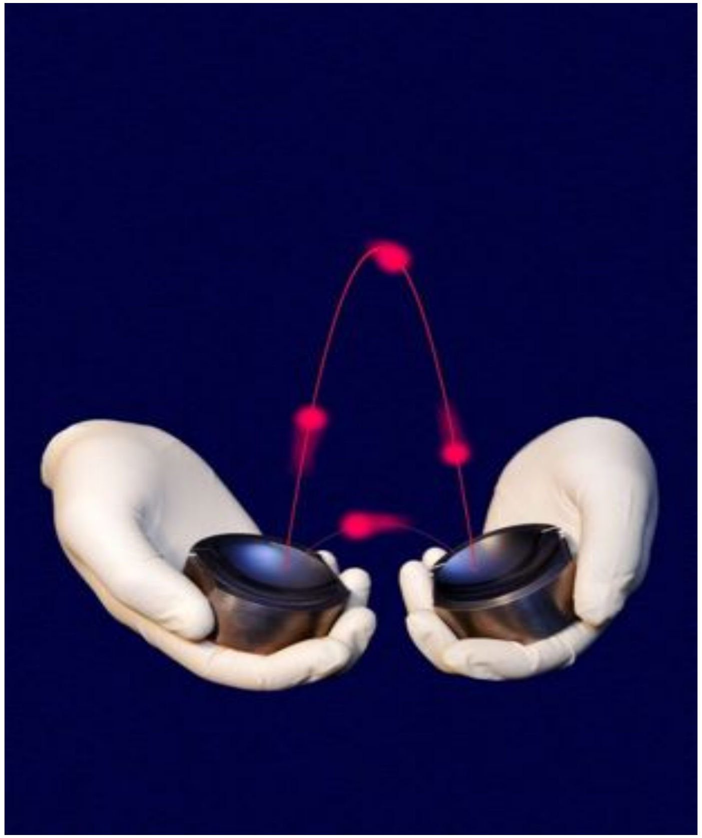
Collège de France abroad Lectures

Quantum information with real or artificial atoms and photons in cavities

Lecture 4:

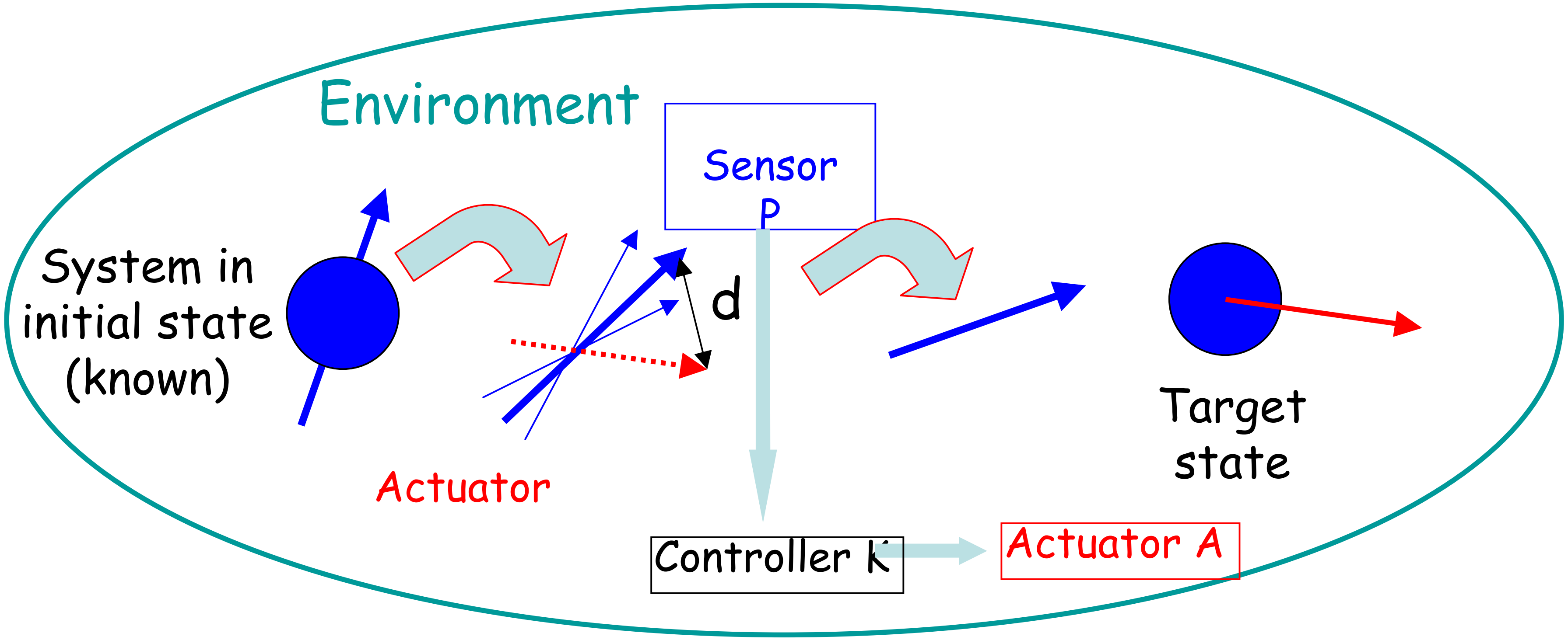
Quantum feedback experiments in Cavity QED



Aim of lecture: illustrate on a Cavity QED example the quantum feedback procedure: how to combine measurements and actuator actions on a quantum system to drive it towards a predetermined state and protect this state agaisnt decoherence

IV-A

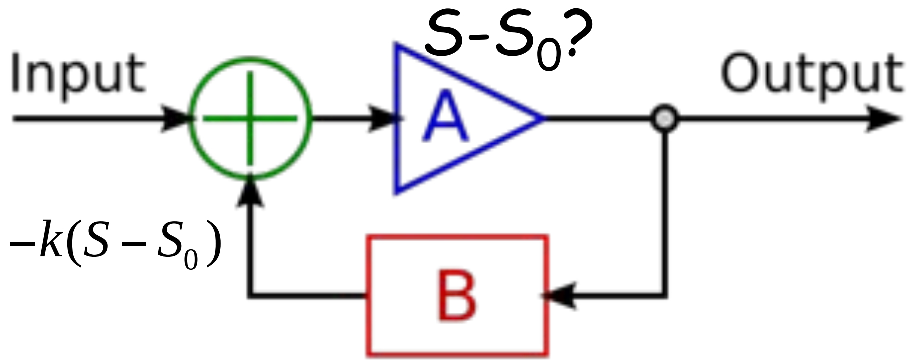
Introduction: principle of quantum feedback



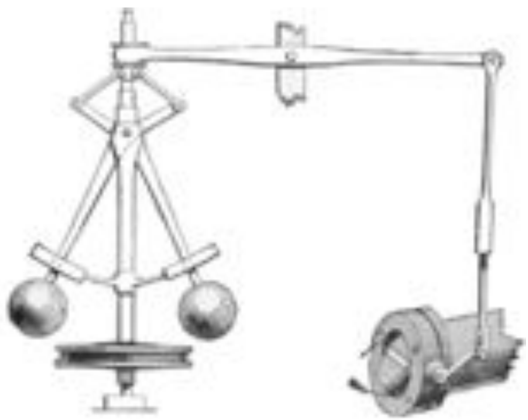
A system S coupled to an environment E is initially in a state $|\psi_i\rangle$. The goal is to drive it to a target state $|\psi_t\rangle$. An actuator A coupled to S transforms its state. Then a sensor (P) performs a measurement sent to a controller (K) which estimates the new state, taking measurement and known effect of environment into account. A distance d to target is computed and K determines the action A should perform to minimize this distance d. Operation repeated in loop until target is reached.

Comparison with classical feedback

A measurement on system is perfomed (A) and result S is compared to a reference value S_0. A feedback signal -k $(S-S_0)$ (where -k is the negative gain of the loop) is applied to the system (B) to bring it closer to the ideal operating point. The device operates in closed feedback loop.



The feedback can be based on an automatic physical effect with a device combining the measurement and the response mechanism: the Watt regulator of the steam machine is a good example. In other cases, the feedback implies two separate ingredients: a reading apparatus which measures the error signal and an actuator of the response, the link between the two being made by a computer (example: speed controller in an automobile).



The extension of these ideas to a quantum system must incorporate an essential element: the measurement has a back-action, independent of any added feedback effect, on the system's state. This quantum back-action must be taken into account to implement the quantum feedback.

Applying quantum feedback to the stabilization of Fock states?

Fock states are interesting examples of non-classical states

They are fragile and lose their non-classicality in time scaling as 1/n.

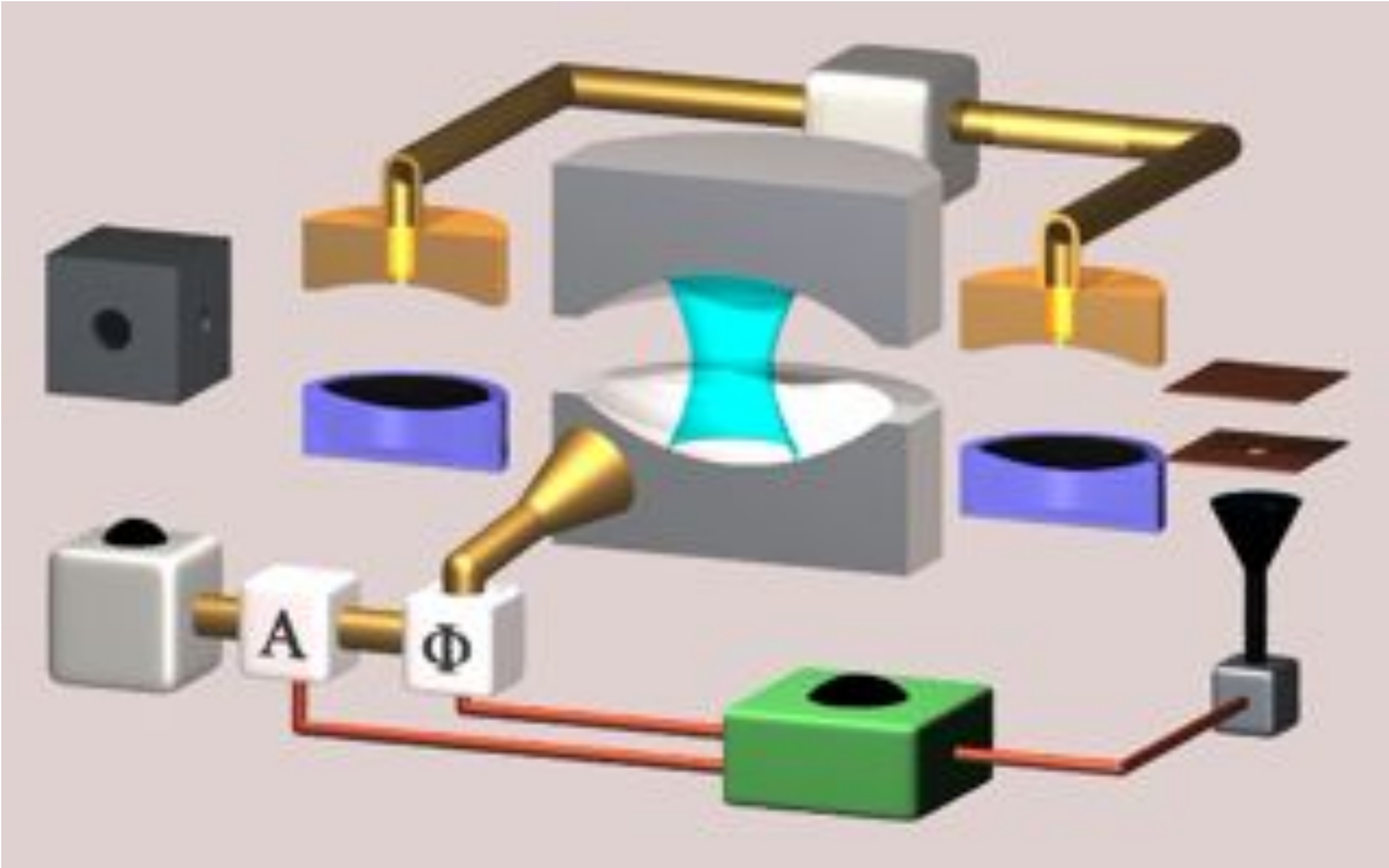
The preparation by projective measurement is random

Is it possible to prepare them in a deterministic way by using quantum feedback procedures?

Can these procedures protect them against quantum jumps (loss or gain of photons)?

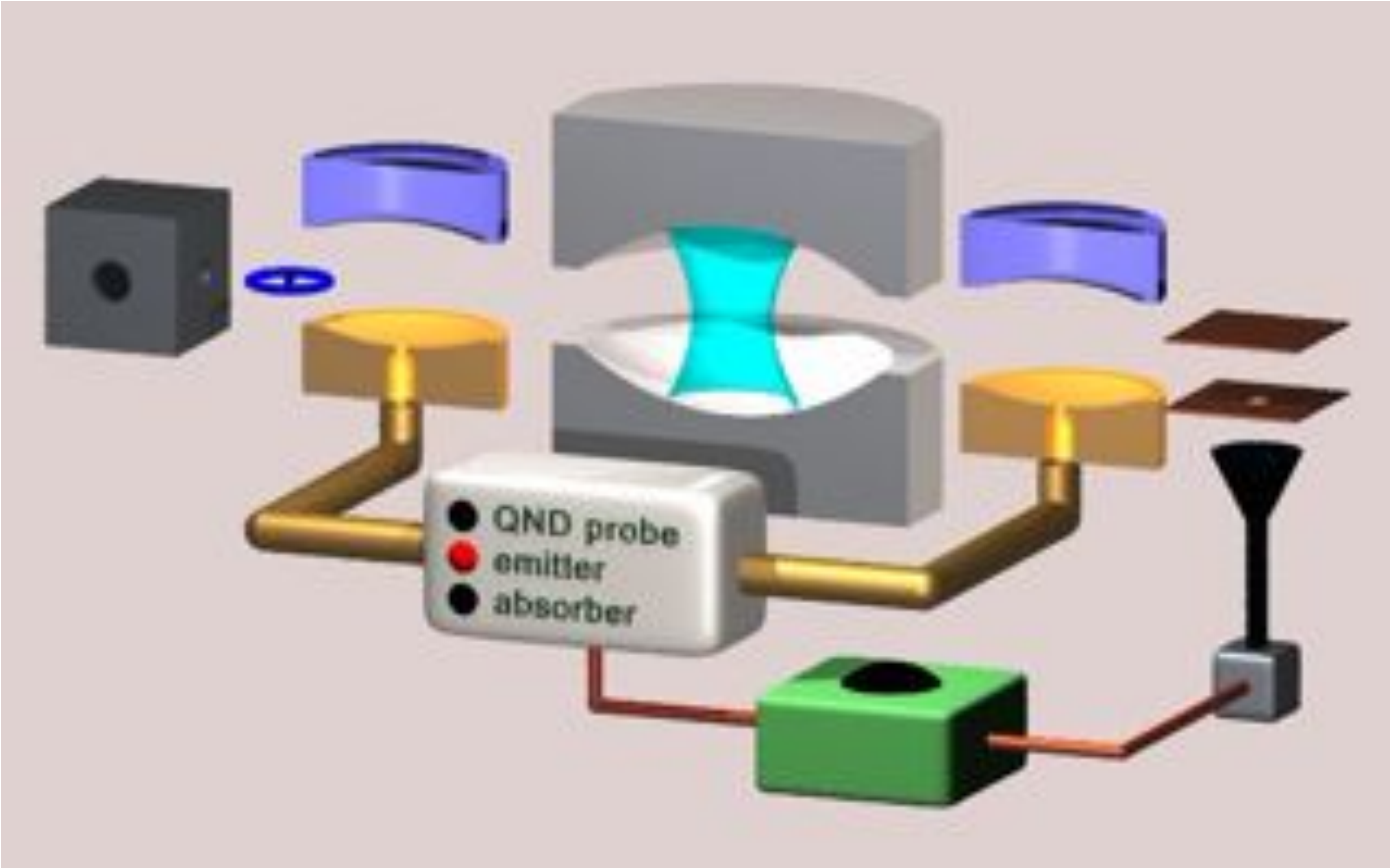
An ideal sensor for these experiments: QND probe atoms measuring photon number by Ramsey interferometry (see lecture 2). This probe leaves the target state invariant!

What kind of actuator? Classical or quantum?



Quantum feedback with classical actuator

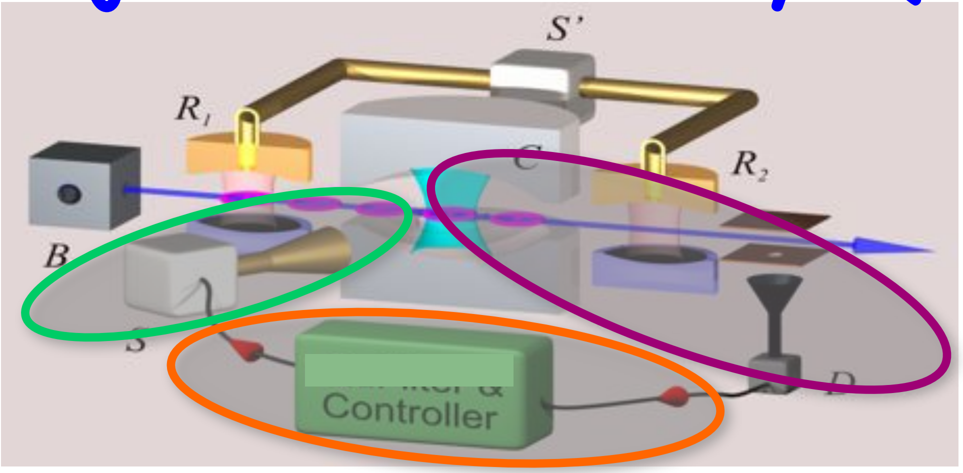
Quantum feedback with quantum actuator; atoms not only probe the field (dispersively), but also emit or absorb photons (resonantly)



IV-B

Quantum feedback by classical field injections

Principle of quantum feedback by field injections in Cavity Quantum electrodynamics



Components of feedback loop

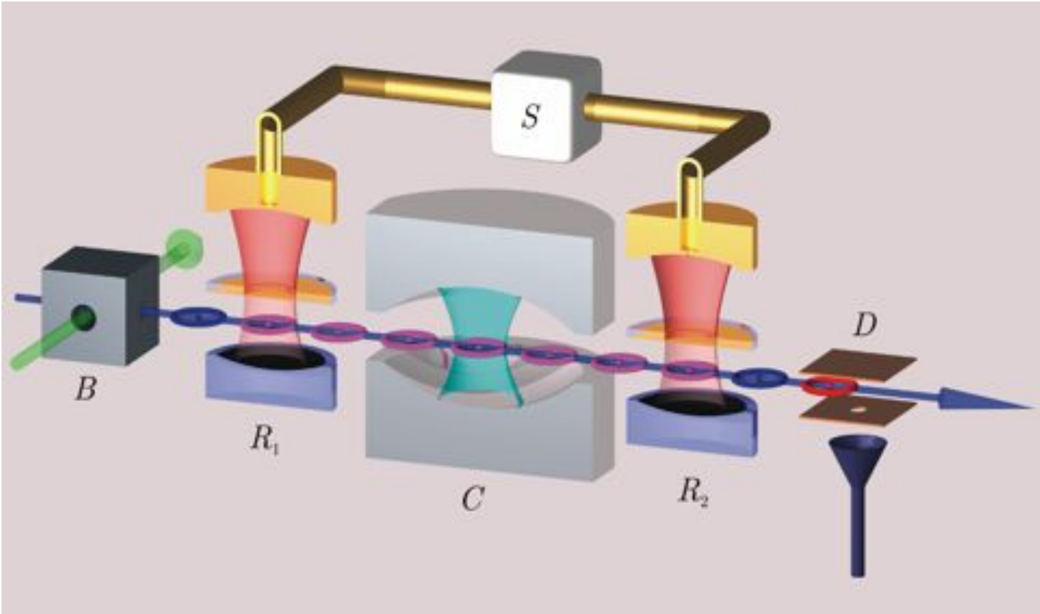
- **Sensor** (quantum "eye"):
  atoms and QND measurements
- **Controller** ("brain"):
  computer
- **Actuator** (classical "hand"):
  microwave injection

Feedback protocol:

- Inject an initial coherent field in C
- Send atoms one by one in Ramsey interferometer
- Detect each atom, projecting field density operator ρ in new state estimated by computer
- Compute displacement α which minimises distance D between target and new state
- Close feedback loop by injecting a coherent field with amplitude α in C
- Repeat loop until reaching D ~ 0.

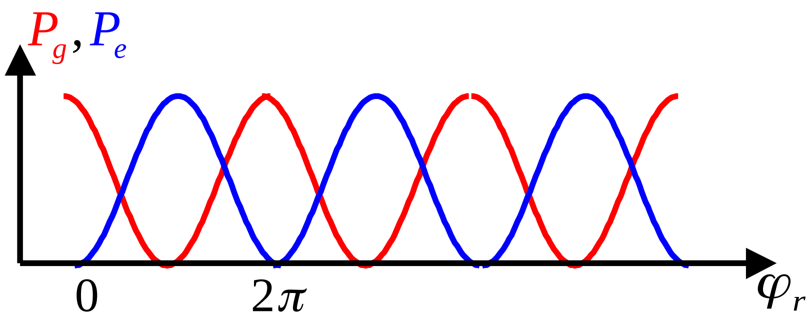
The CQED Ramsey Interferometer

The Ramsey interferometer is made of two auxiliary cavities R_1 et R_2 sandwiching the cavity C containing the field to be measured. The atom with two levels g and e (qubit in states j=0 and j=1 respectively), prepared in e, is submitted to classical $\pi/2$ pulses in R_1 and R_2, the second having a φ_r phase difference with the first. The probabilities to detect the atom in g (j=0) and e (j=1) when C is empty are:



$$P_j = \cos^2 \frac{(\varphi_r - j\pi)}{2} \quad ; \quad j = 0,1$$

The P_j probabilities oscillate ideally between 0 and 1 with opposite phases when φ_r is swept (Ramsey fringes).



Single atom detection (see lecture 2)

Initial state

$$(j = e, g) \quad \rho_{\mathrm{proj}} = \frac{M_j \, \rho \, M_j^\dagger}{\mathrm{Tr}(M_j \, \rho \, M_j^\dagger)}$$

State after projection

direction of measurement

$$M_e = \sin\left(\frac{\phi_r + \phi(N)}{2}\right)$$

2 operators corresponding to the 2 possible outcomes

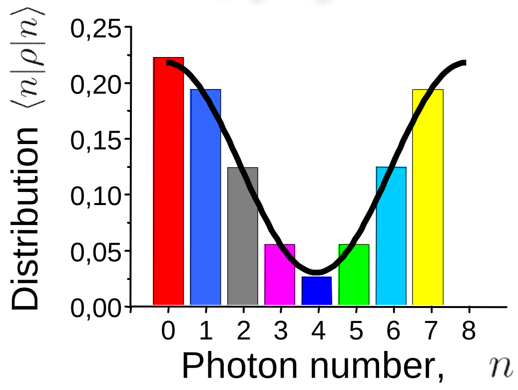
Photon number operator

phaseshift per photon

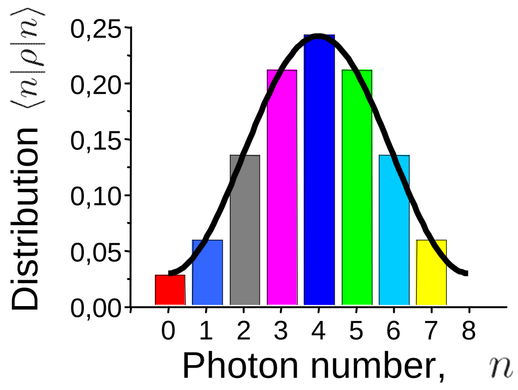
$$M_g = \cos\left(\frac{\phi_r + \phi(N)}{2}\right)$$

Atomic detection changes the photon number distribution

ρ_{proj}



atom in $|e\rangle$



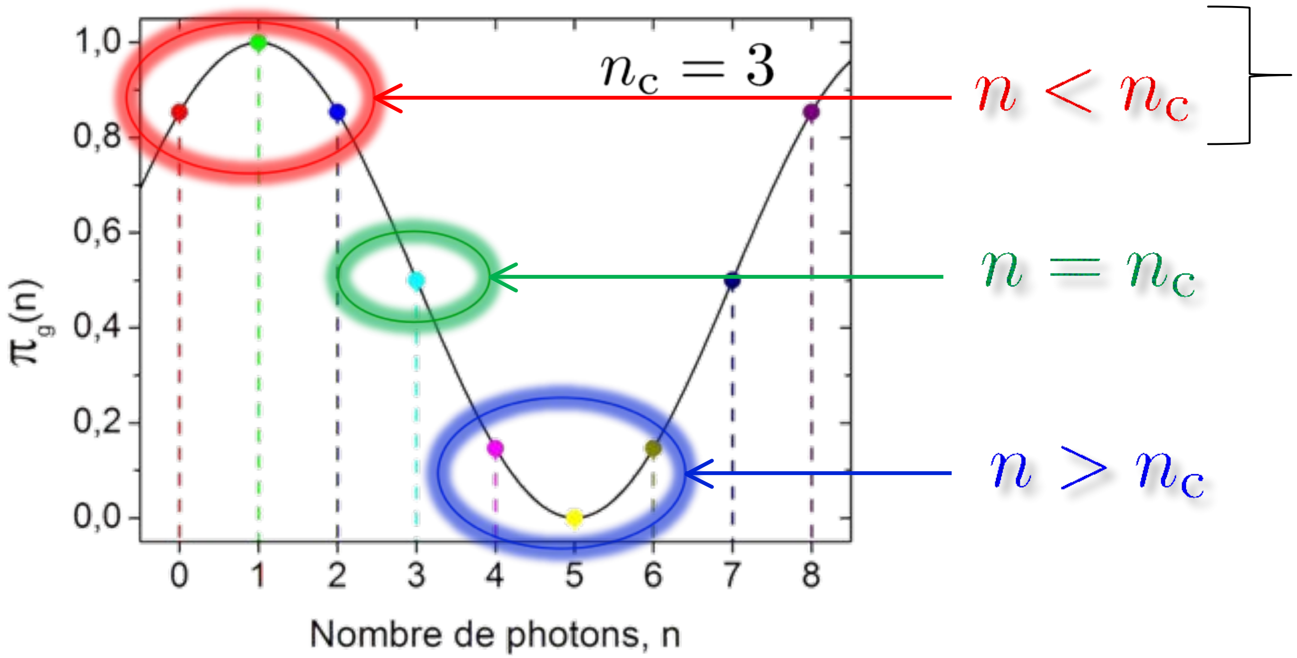
atom in $|g\rangle$

Probe : weak measurement

Fixing the parameters of experiment

$$M_e = \sin\left(\frac{\phi_r + \phi(N)}{2}\right) \qquad M_g = \cos\left(\frac{\phi_r + \phi(N)}{2}\right)$$

- Phaseshift per photon : $\phi_0 = \pi/4 \qquad (\phi(n) = n\phi_0)$
- Ramsey phase : $\phi_r = \pi/2 - \phi(n_c)$



Three well

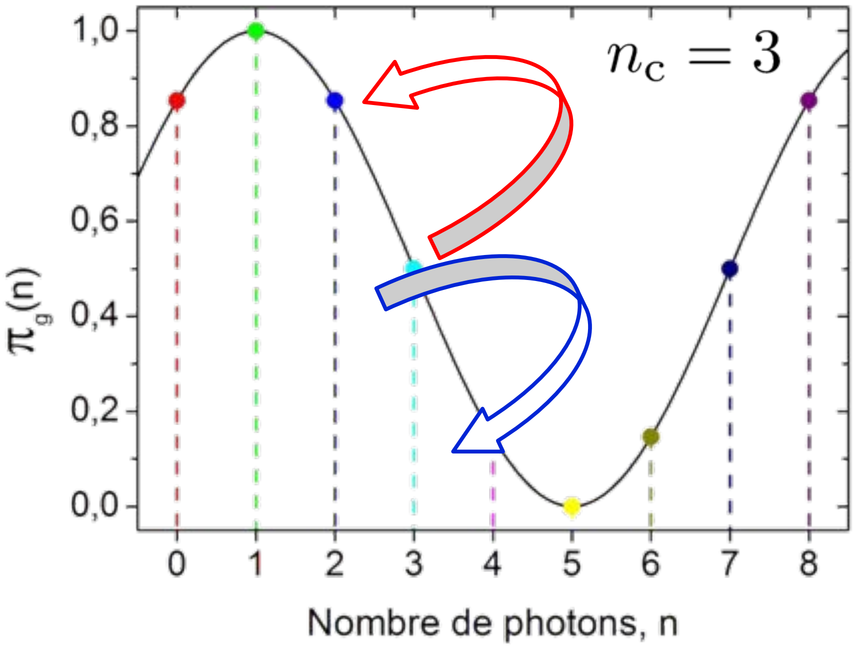
distinct sets

Probe : weak measurement

Fixing the parameters of experiment

$$M_e = \sin\left(\frac{\phi_r + \phi(N)}{2}\right) \qquad M_g = \cos\left(\frac{\phi_r + \phi(N)}{2}\right)$$

- Phaseshift per photon : $\phi_0 = \pi/4 \quad (\phi(n) = n\phi_0)$
- Ramsey phase: $\phi_r = \pi/2 - \phi(n_c)$



Quantum jumps well detected

- $|n_c\rangle \longleftrightarrow |n_c - 1\rangle$
- $|n_c\rangle \longleftrightarrow |n_c + 1\rangle$

Controler : real time estimation of field state

Before weak measurement, field described by density matrix ρ

- Weak measurement

Detected atom : outcome $|j = e, g\rangle$

$$\rho_{\mathrm{proj}} = \frac{M_j\,\rho\,M_j^\dagger}{\mathrm{Tr}(M_j\,\rho\,M_j^\dagger)}$$

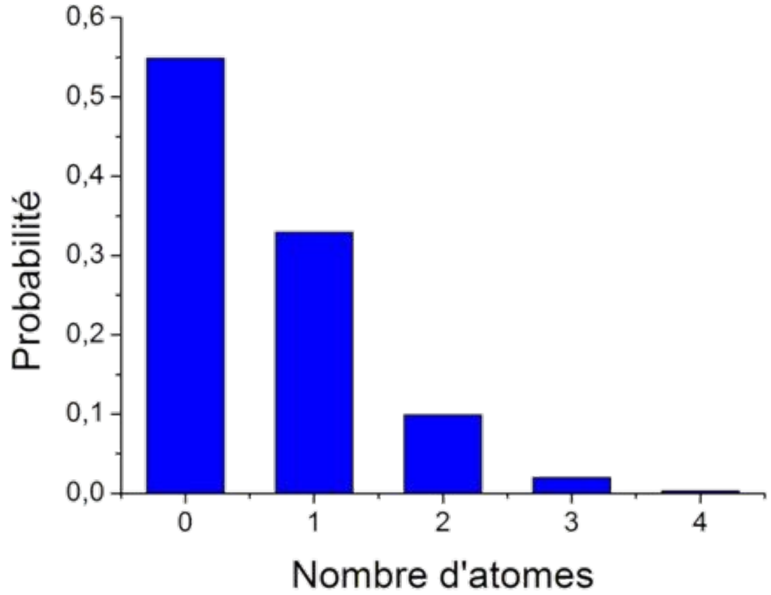
« Ideal » situation: does not take into account the imperfections of experimental set-up !

Controler : field state estimation

Difficulty : atomic source is not deterministic

Poisson law for atom number per sample with average : $n_a \approx 0{,}6$ atom

$$P_a(n) = \mathrm{e}^{-n_a} \frac{n^{n_a}}{n!}$$



Most probable : no atom in sample

Two atoms possible

New POVM operators when 2 atoms detected

$$M_{ee} = M_e \times M_e \qquad M_{gg} = M_g \times M_g$$

$$M_{eg} = \sqrt{2}\, M_e \times M_g$$

Controler : field state estimation

Difficulty : imperfect apparatus

- Detection efficiency : $\epsilon_d \approx$ 35 % of atoms are counted

Unread measurements:

$$\rho_{\text{proj}} = M_e \, \rho \, M_e^\dagger + M_g \, \rho \, M_g^\dagger$$

Controler : field state estimation

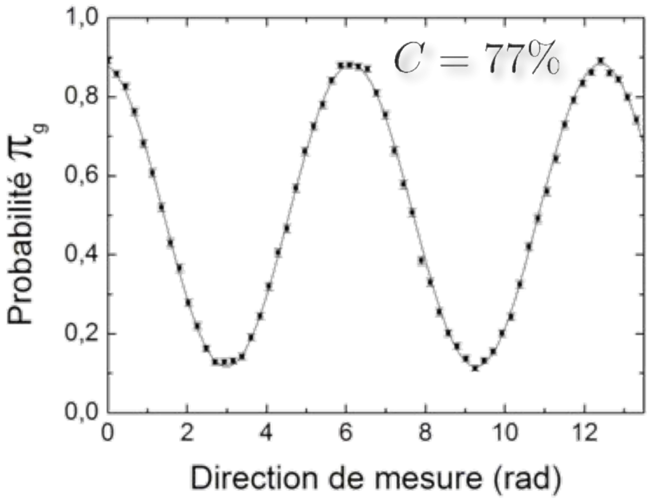
Difficulty : imperfect apparatus

- Detection efficiency : $\epsilon_d \approx$ 35 % of atoms are counted

Unread measurement

$$\rho_{\mathrm{proj}} = M_e\,\rho\,M_e^\dagger + M_g\,\rho\,M_g^\dagger$$

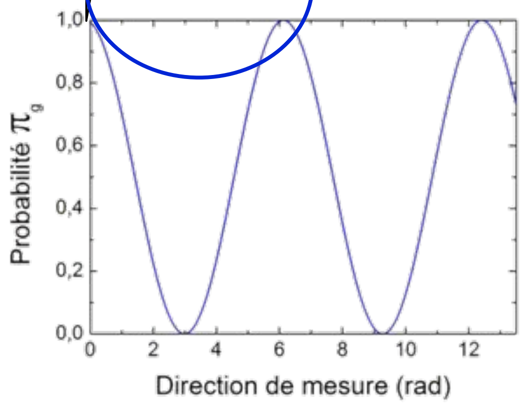
- Limited interferometer contrast

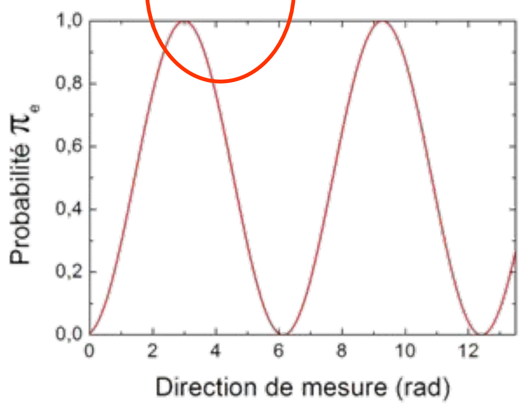


Detection errors

proportion of atoms in $|e\rangle$ detected in $|g\rangle$

$$1 - \eta_g = 88,5\% \quad + \quad \eta_e = 11,5\%$$





Controler : field state estimation

Difficulty : imperfect apparatus

- Poisson statistics
- Detection efficiency
- Detection errors

Assume 1 atom detected in state $|e\rangle$

- Was really the atom in this state? $|e\rangle$ or $|g\rangle$?
- Was a second atom missed ?
- If so, in which state was it ? $|e\rangle$ or $|g\rangle$?

$$\rho_{\mathrm{proj}} = \frac{M_e \rho M_e^\dagger}{\mathrm{Tr}\left(M_e \rho M_e^\dagger\right)} \equiv \rho_e$$

$$\rho_{\mathrm{proj}} = p(e|e^d)\rho_e + p(g|e^d)\rho_g + p(ee|e^d)\rho_{ee} + p(eg|e^d)\rho_{eg} + p(gg|e^d)\rho_{gg}$$

All conditional probabilities given by Bayes law, knowing calibrated imperfections

I. Dotsenko *et al.*, Phys. Rev. A 80, 013805 (2009)

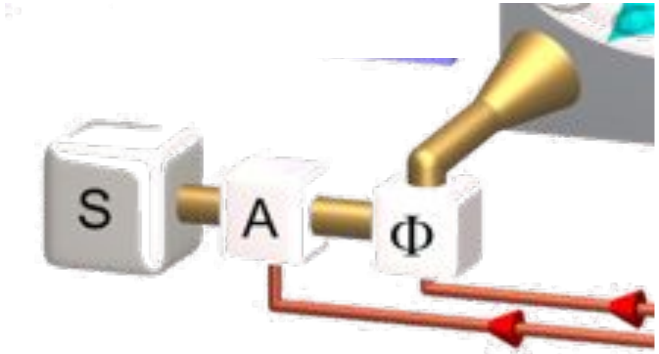
Actuator : field displacement

Change photon number distribution *via* field displacement

Displacement operator $D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a)$: *injection of coherent field in cavity*

$$\rho_{\rm disp} = D(\alpha)\,\rho\,D(-\alpha) \equiv \mathbf{D}_\alpha \rho$$

amplitude of displacement : complex amplitude of microwave pulse



In experiment :

- α real only
- phase is chosen to be 0 or π , with respect to initial field (fixing sign of displacement)
- Modulus |α| is controled *via* duration of microwave pulse

$$\alpha_{\max} = 0,1 \qquad t_{\max} = 60\ \mu\text{s}$$

$$\alpha_{\min} = 0,001 \qquad t_{\min} = 0,6\ \mu\text{s}$$

Controler : computing optimal displacement

Choosing displacement amplitude : moving field closer to target

Minimise proper distance to desired number state

❖ A straightforward definition :

$$d_F(\rho,\rho_c) = 1 - \langle n_c|\rho|n_c\rangle$$

Fidelity with respect to target

Drawback : Other Fock states are undistinguishable

$$d_F(|n\rangle\langle n|\,,\rho_c) = 1 \quad \text{for} \quad n \neq n_c \quad (n = n_c \pm 1, n \gg n_c, \ldots)$$

❖ A better definition :

$$d(\rho,\rho_c) = \sum_n \Gamma_n^{(n_c)} \langle n|\rho|n\rangle$$

Controler : computing optimal displacement

Choosing displacement amplitude : moving field closer to target

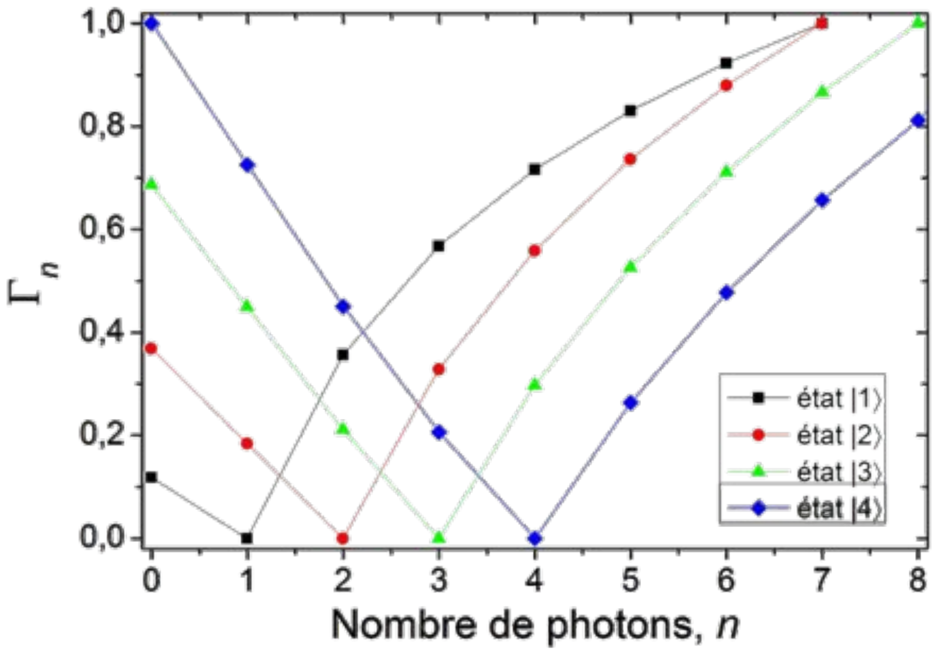
Minimise proper distance to desired number state

- A straightforward definition : :

$$d_F(\rho, \rho_c) = 1 - \langle n_c|\rho|n_c\rangle$$

- A better definition :

$$d(\rho, \rho_c) = \sum_n \Gamma_n^{(n_c)} \langle n|\rho|n\rangle$$



The further n is from n_c ,
the larger the distance to the target!

$$\Gamma_{n_c}^{(n_c)} = 0$$

Controler : computing optimal displacement

- Minimisation : $$\alpha = \arg\min_{\alpha \in \mathbb{R}} d(\mathbf{D}_\alpha \rho, \rho_c)$$

⟹ Very costly in computing time !

- To speed up the process : restrict to small displacement amplitudes

⟹ Define a maximum amplitude : $\alpha_{max} = 0{,}1$

⟹ Behaviour of $d(\mathbf{D}_\alpha \rho, \rho_c)$ around $\alpha = 0$?

$$d(\mathbf{D}_\alpha \rho, \rho_c) = d(\rho, \rho_c) - a_1(\rho)\alpha - a_2(\rho)\frac{\alpha^2}{2}$$

Coefficients $\Gamma_n^{(n_c)}$ chosen so that :

- If $\rho = |n_c\rangle\langle n_c| = \rho_c$ ⟹ $d(\mathbf{D}_\alpha \rho, \rho_c)$ is minimum at $\alpha=0$

  $(a_1(\rho_c) = 0 \quad a_2(\rho_c) < 0)$

- If $\rho = |n \neq n_c\rangle\langle n \neq n_c|$ ⟹ $d(\mathbf{D}_\alpha \rho, \rho_c)$ is maximum at $\alpha=0$

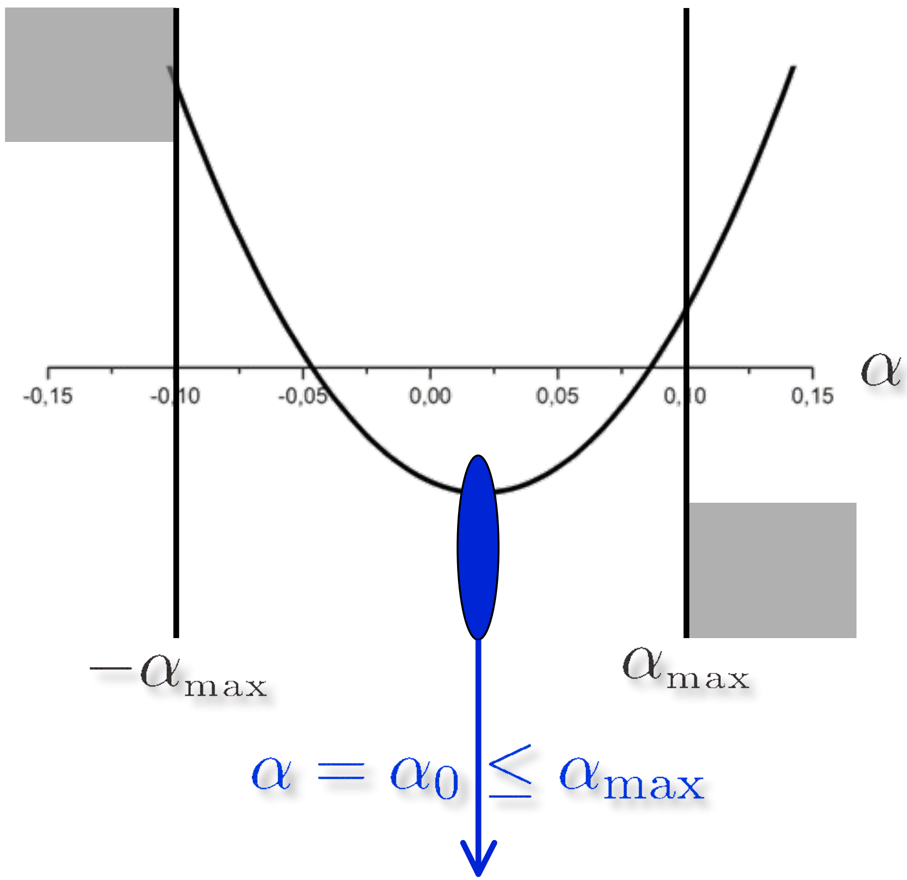
  $(a_1(|n\rangle\langle n|) = 0 \quad a_2(|n\rangle\langle n|) > 0)$

Controler : computing optimal displacement

$$d(\mathbf{D}_\alpha \rho, \rho_c) = d(\rho, \rho_c) - a_1(\rho)\alpha - a_2(\rho)\frac{\alpha^2}{2}$$

- Control law : studying the function

It has a local minimum on [$-\alpha_{max}$, + α_{max}]

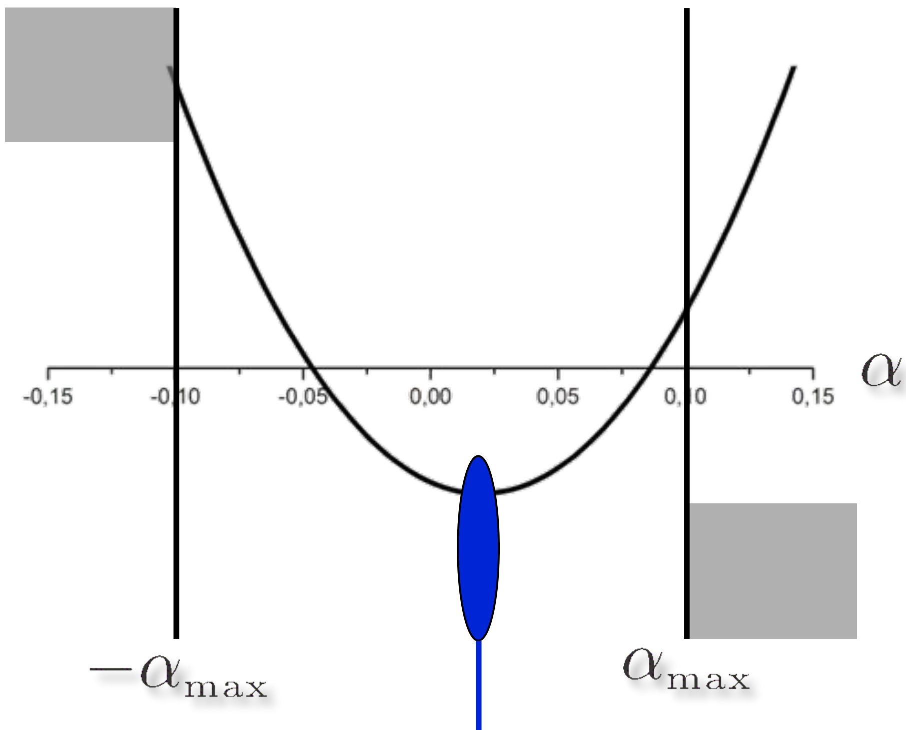


Controler : computing optimal displacement

$$d(\mathbf{D}_\alpha \rho, \rho_c) = d(\rho, \rho_c) - a_1(\rho)\alpha - a_2(\rho)\frac{\alpha^2}{2}$$

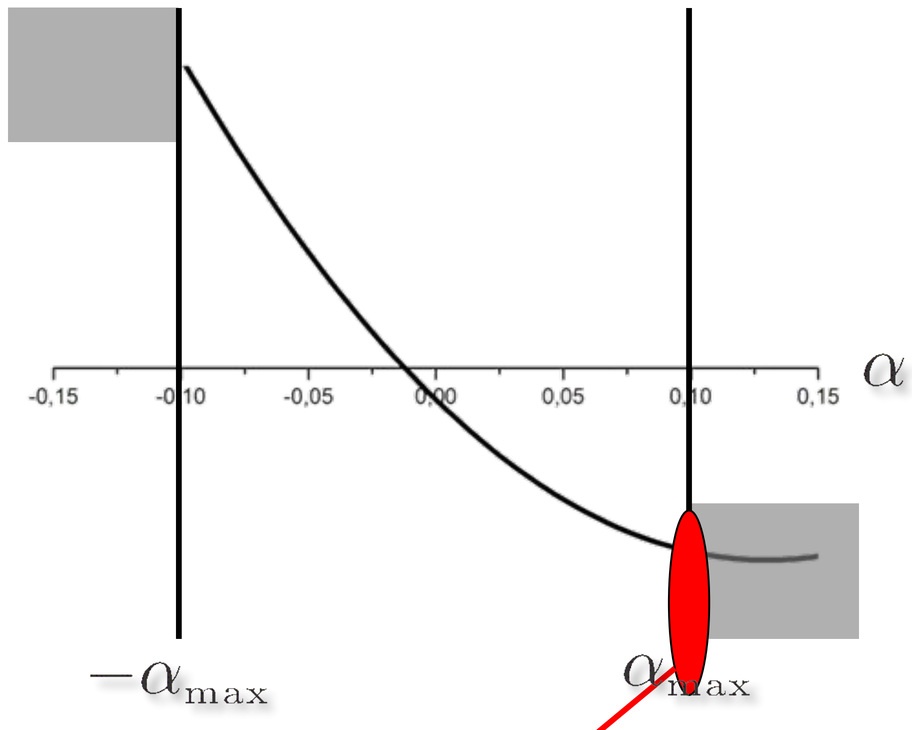
•Control law : studying the function $\alpha \mapsto d(\mathbf{D}_\alpha \rho, \rho_c)$

If local minimum on [$-\alpha_{max}$, + α_{max}]



$$\alpha = \alpha_0 \leq \alpha_{\max}$$

If Local minimum outside [$-\alpha_{max}$, + α_{max}]



$$\alpha = +\alpha_{\max}$$

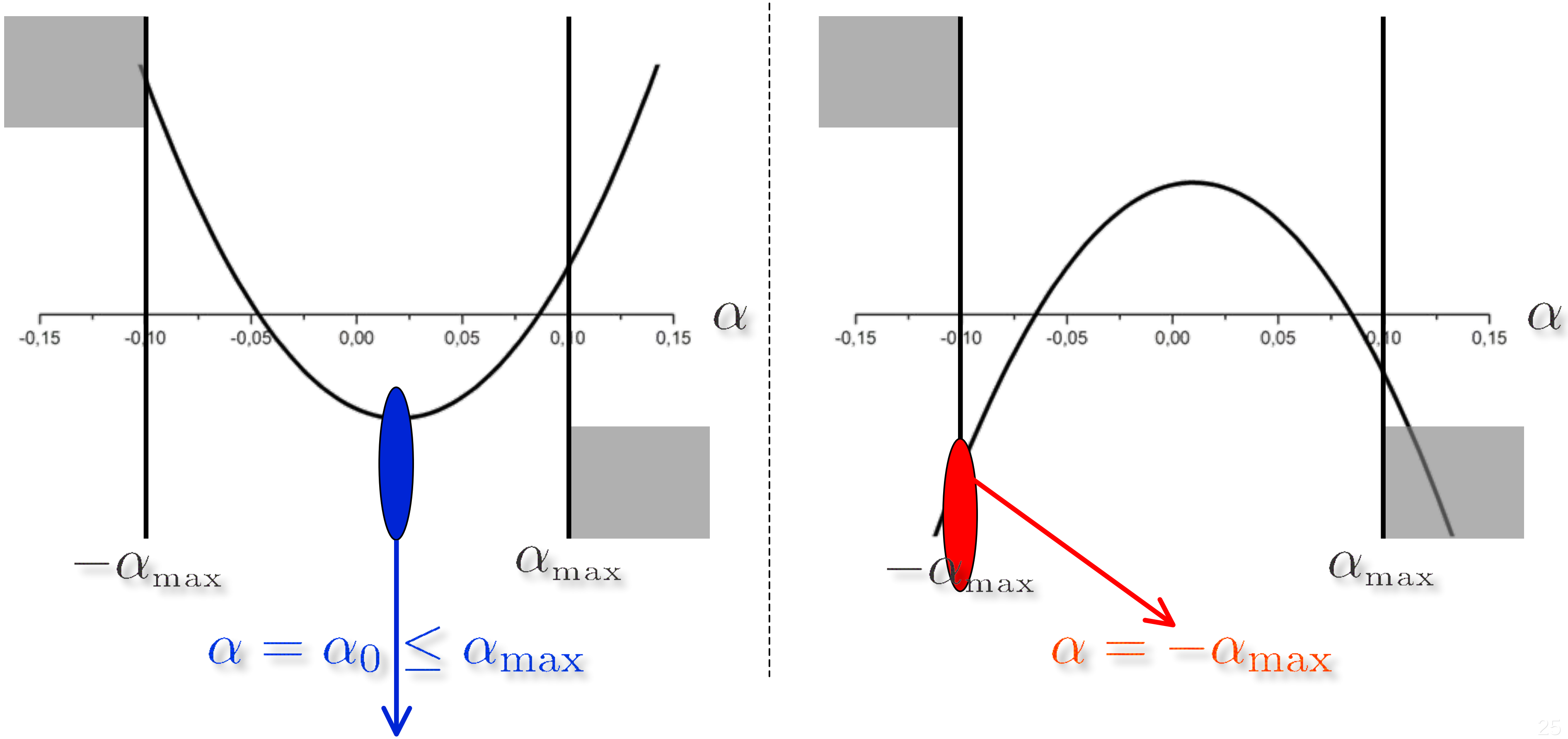
Controler : computing optimal displacement

$$d(\mathbf{D}_\alpha \rho, \rho_c) = d(\rho, \rho_c) - a_1(\rho)\alpha - a_2(\rho)\frac{\alpha^2}{2}$$

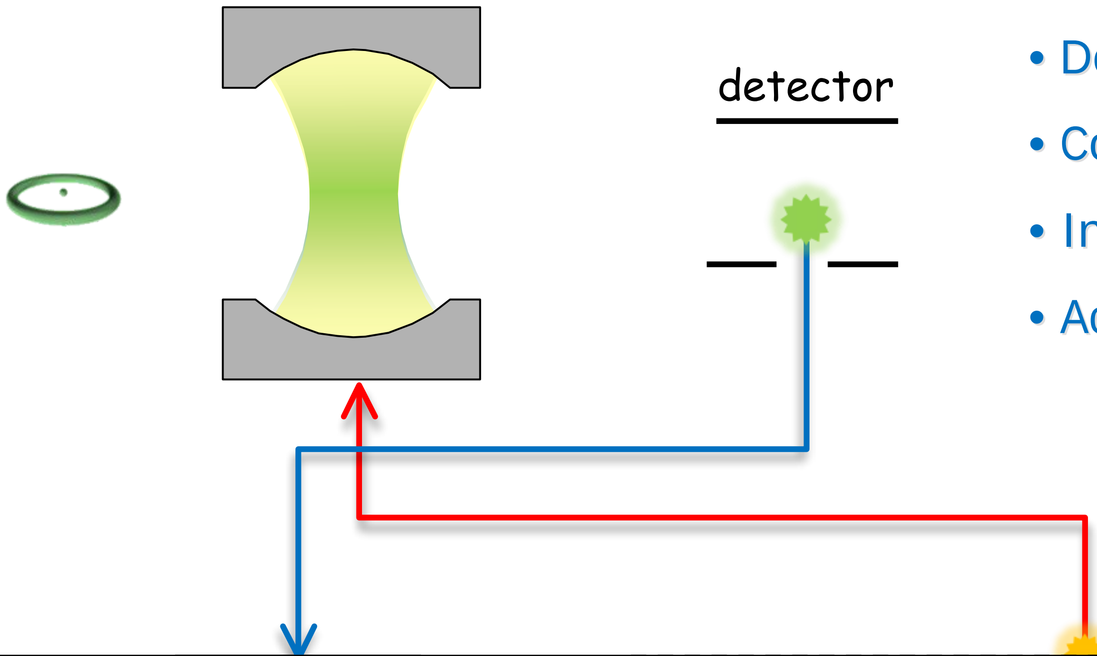
•Control law : studying the function $\alpha \mapsto d(\mathbf{D}_\alpha \rho, \rho_c)$ ρ

If local minimum on [-α_{max} , + α_{max}]

If Local maximum on [-α_{max} , + α_{max}]



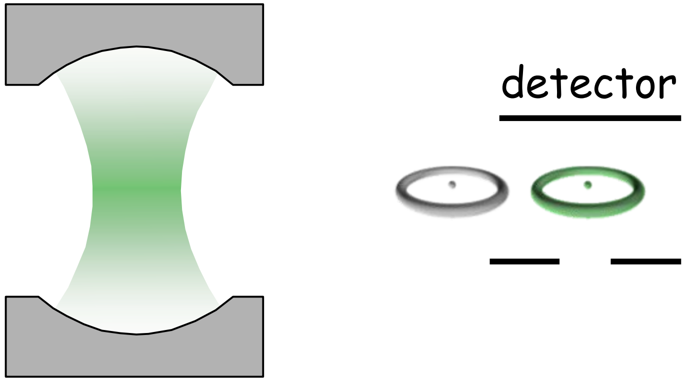
Summing it up: the feedback loop



- Detection of atomic sample
- Computing optimal displacement
- Injecting control field
- Accounting for relaxation

ρ^k ρ^{k+1}	Atomic detection $\vert j\rangle$	ρ^k_{proj} $=\mathbf{M}_j\rho^k$	Computing optimal displacement $\alpha = \alpha_{\mathrm{optimum}}$	$\mathcal{D}(\alpha)$	ρ^k_{disp} $=\mathbf{D}_\alpha\rho^k_{\mathrm{proj}}$	Relaxation T_a $= 82\ \mu\mathrm{s}$

Summing it up: the feedback loop



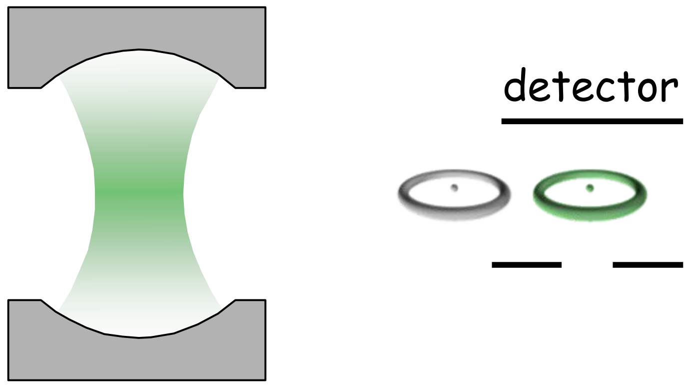
- Detection of atomic sample
- Computing optimal displacement
- Injecting control field
- Accounting for relaxation

Speed requirement : next atom follows after 82 µs !

Computation must take < 80 µs

ρ^{k+1}	Atomic detection $\vert j\rangle$	$\rho^k_{\mathrm{proj}} = \mathbf{M}_j \rho^k$	Computing optimal displacement $\alpha = \alpha_{\mathrm{optimum}}$	$\mathcal{D}(\alpha)$	$\rho^k_{\mathrm{disp}} = \mathbf{D}_\alpha \rho^k_{\mathrm{proj}}$	Relaxation $T_a = 82$ µs

Summing it up: the feedback loop



Simplifying computation

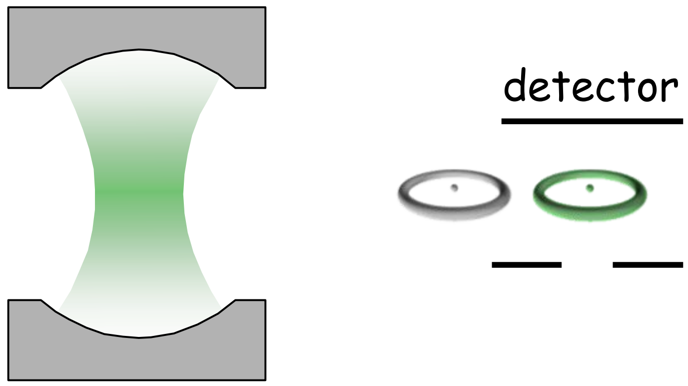
- Real symmetrical matrices
- Developing D_α to 2^{nd} order
- Finite size Hilbert space

Speed requirement : next atom follows after 82 µs !

Computation must take < 80 µs

ρ^{k+1}	Atomic detection $\lvert j\rangle$	$\rho^k_{\text{proj}} = \mathbf{M}_j \rho^k$	Computing optimal displacement $\alpha = \alpha_{\text{optimum}}$	$\mathcal{D}(\alpha)$	$\rho^k_{\text{disp}} = \mathbf{D}_\alpha \rho^k_{\text{proj}}$	Relaxation $T_a = 82\ \mu\text{s}$

Summing it up: the feedback loop



Simplifying computation

- Real symmetrical matrices
- Developing D_α to 2^{nd} order
- Finite size Hilbert space

Speed requirement: next atom follows after 82 µs !

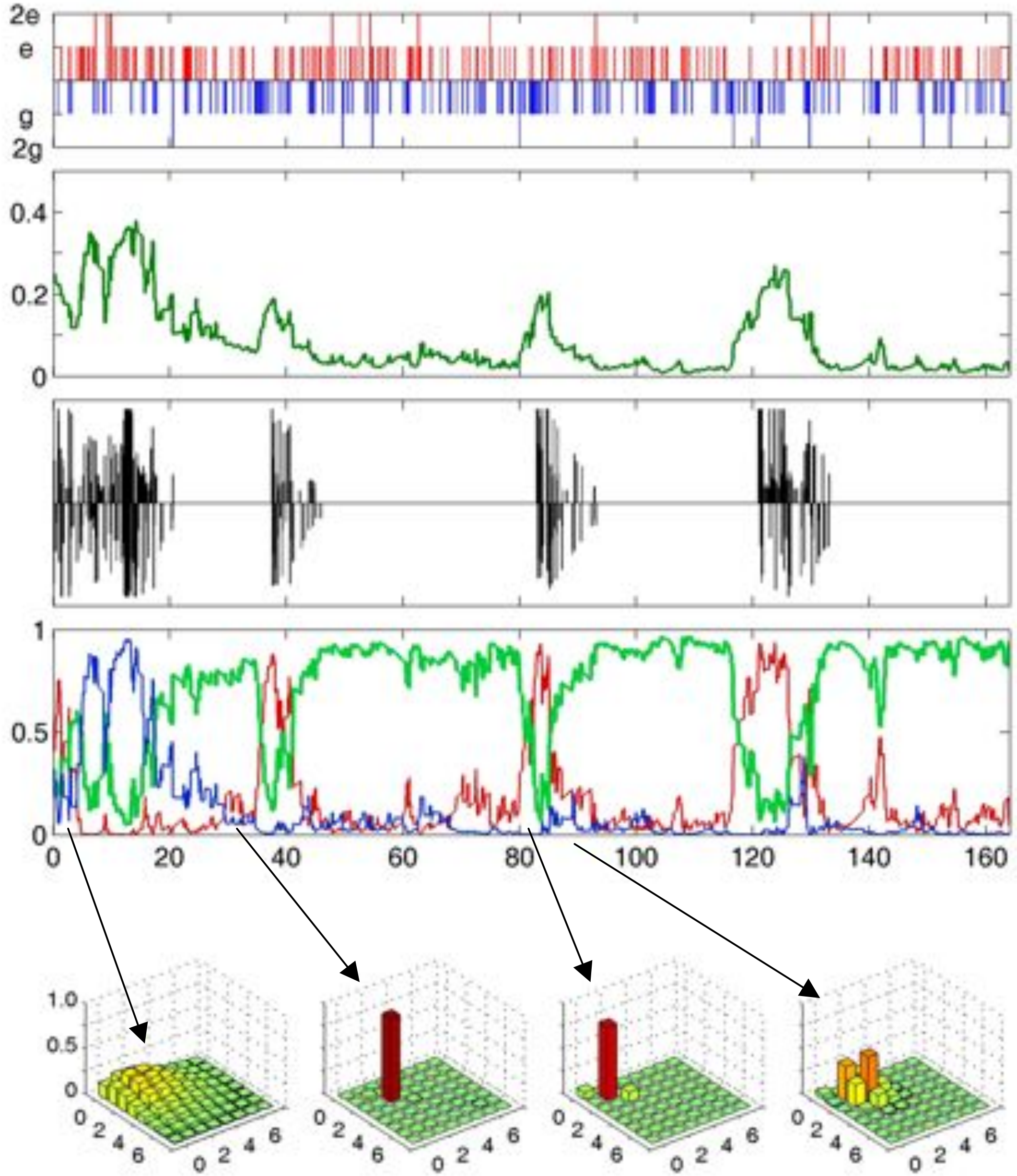
Computation must take < 80 µs

A dedicated control computer

(ADwin Pro-II system)

- Estimates in real time the field state
- Controls microwave injection and atom detection
- Very precise and short response time : ~ (300 ± 30) ns
- Clock cycle : ~ 3,33 ns multiplying two numbers : ~ 2 cycles
- typically : with a Hilbert space truncated at n=9
- Clock cycle : ~ 3,33 ns operations per second : ~ 150 Mflops

n_t=2 target



Raw detection

Distance to target (d)

Actuator injection amplitudes

Estimated photon number probabilities: $P(n=n_t)$, $P(n<n_t)$, $P(n>n_t)$

Estimated density operator

n_t=3 target

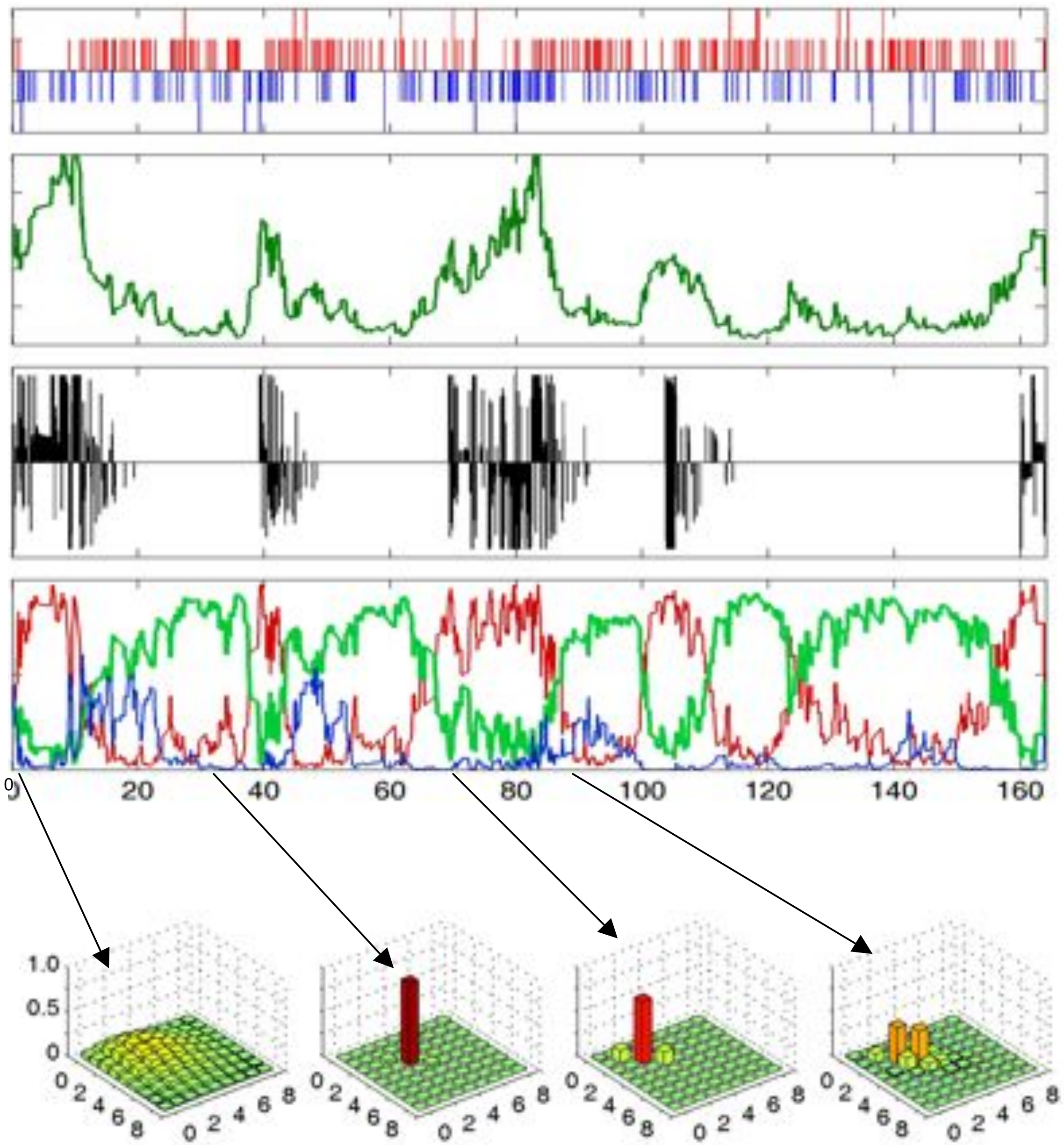
Raw detection

Distance to target

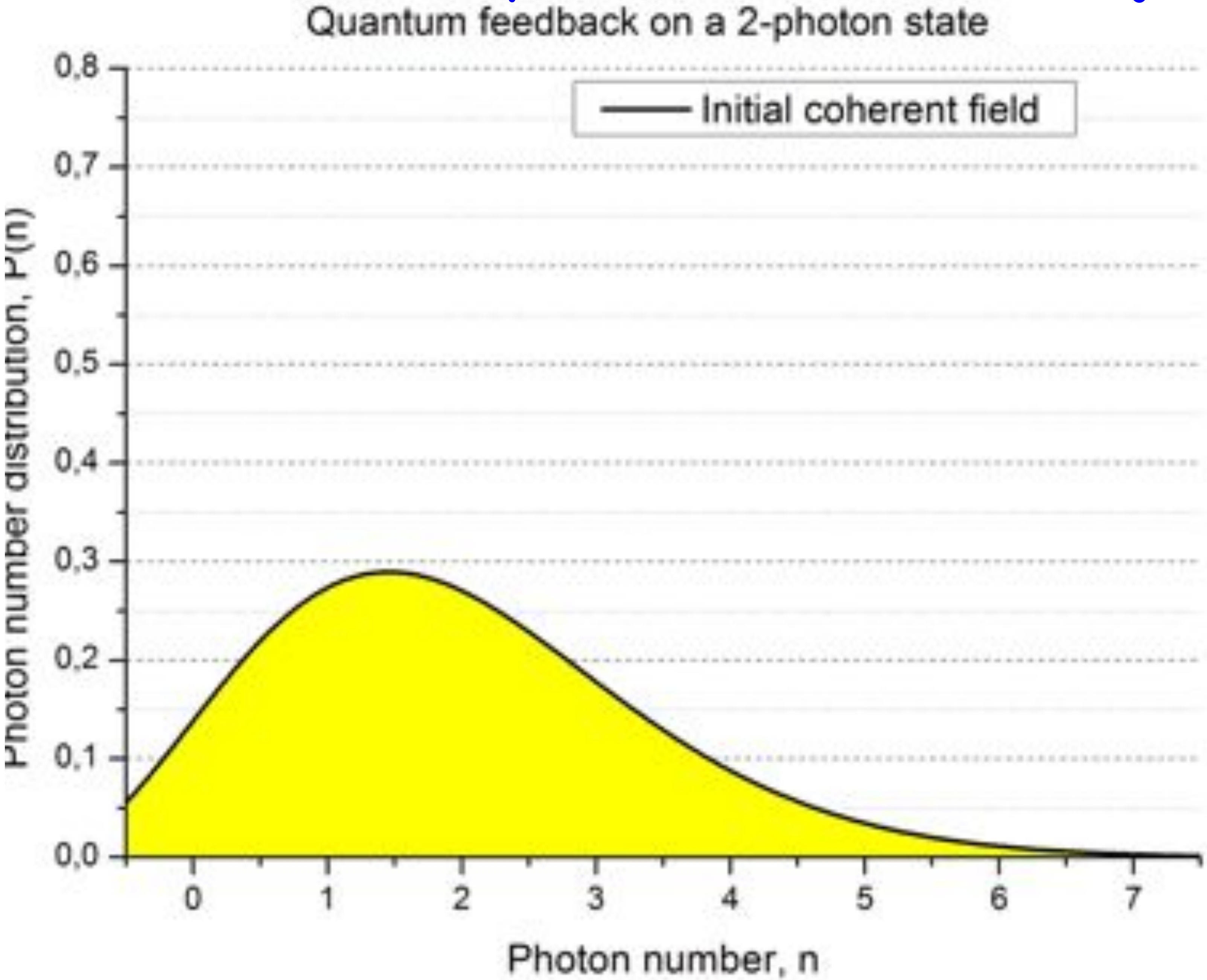
Actuator injection amplitudes

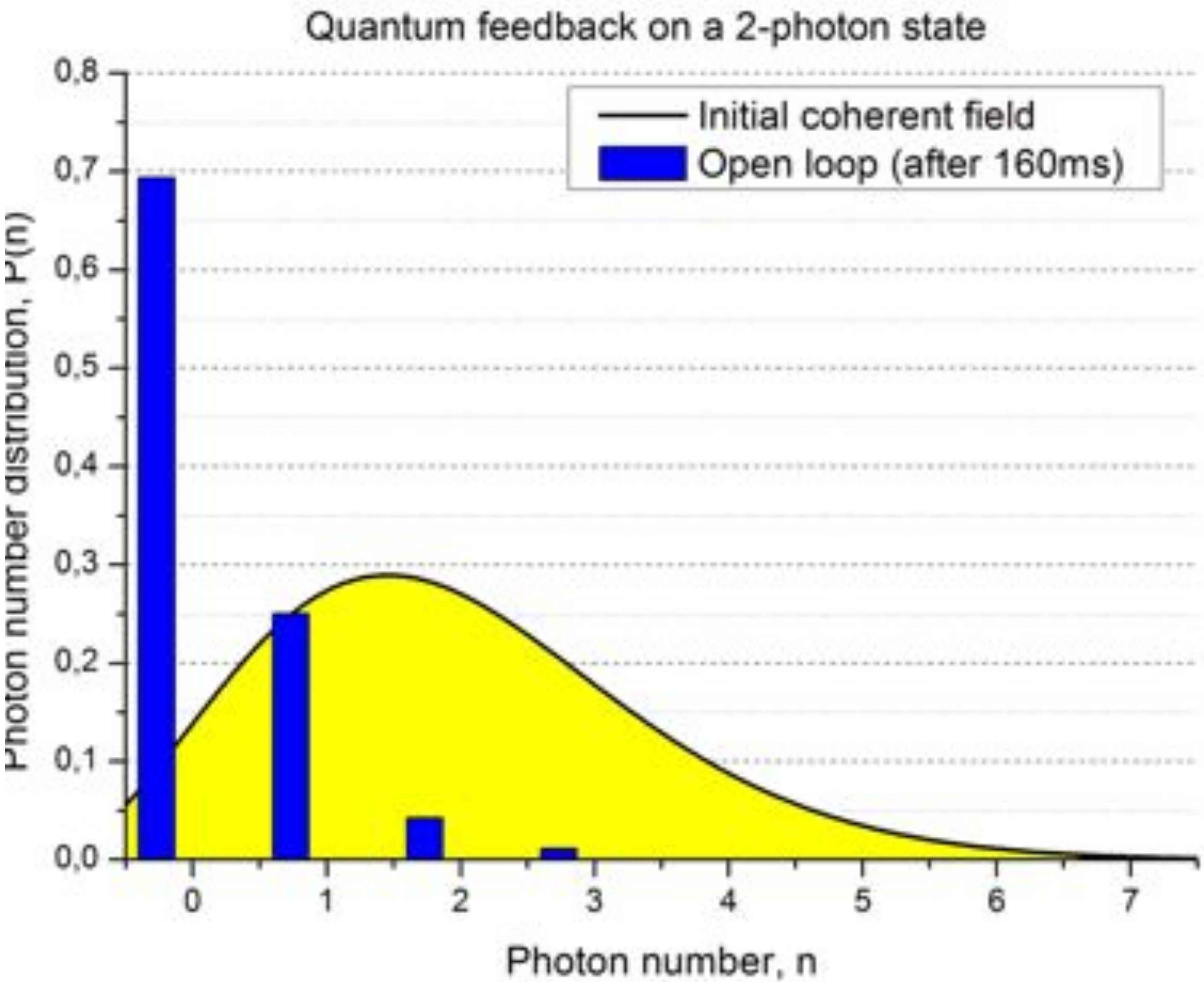
Estimated photon number probabilities:
$P(n=n_t)$,
$P(n<n_t)$, $P(n>n_t)$

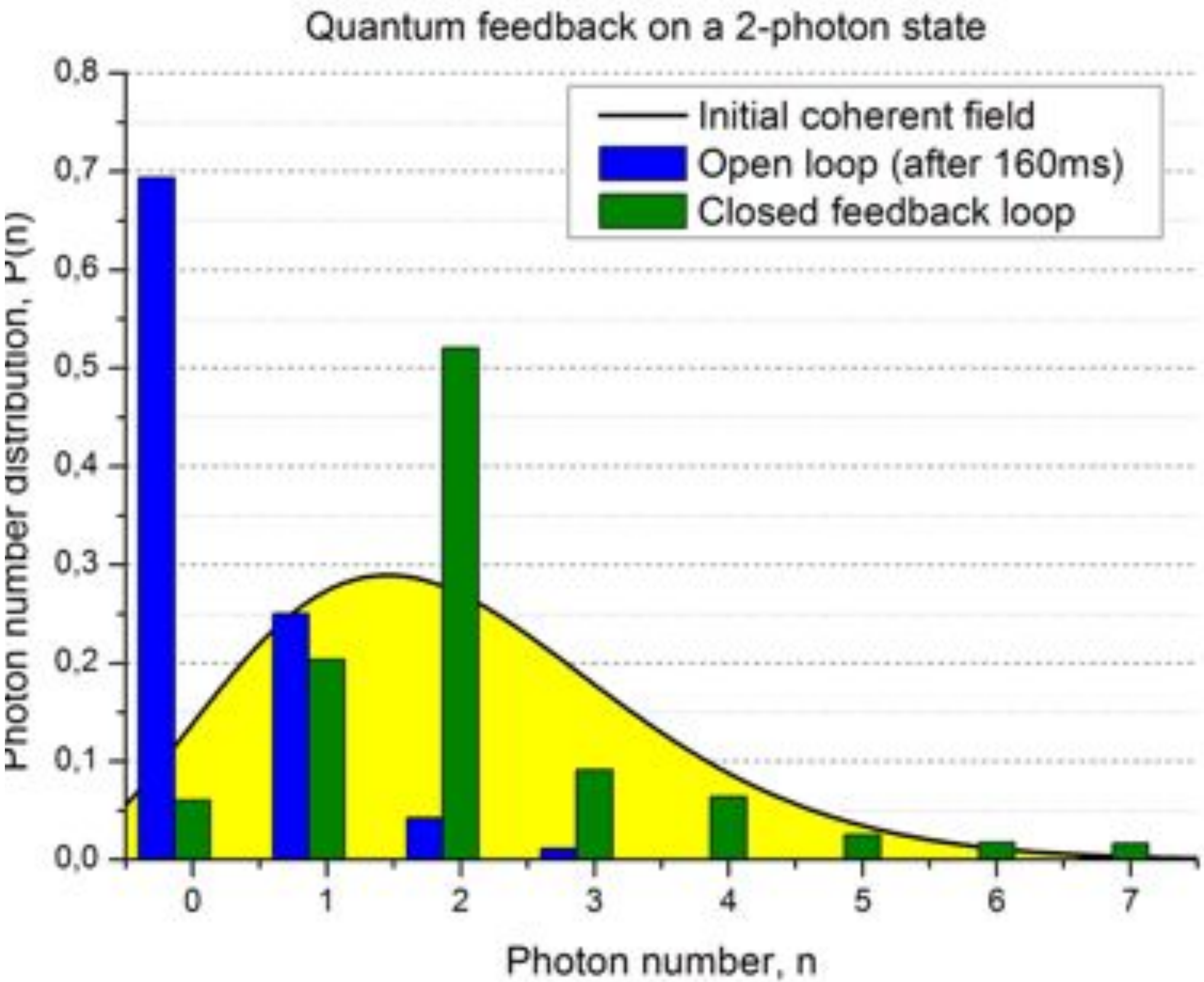
Estimated density operator

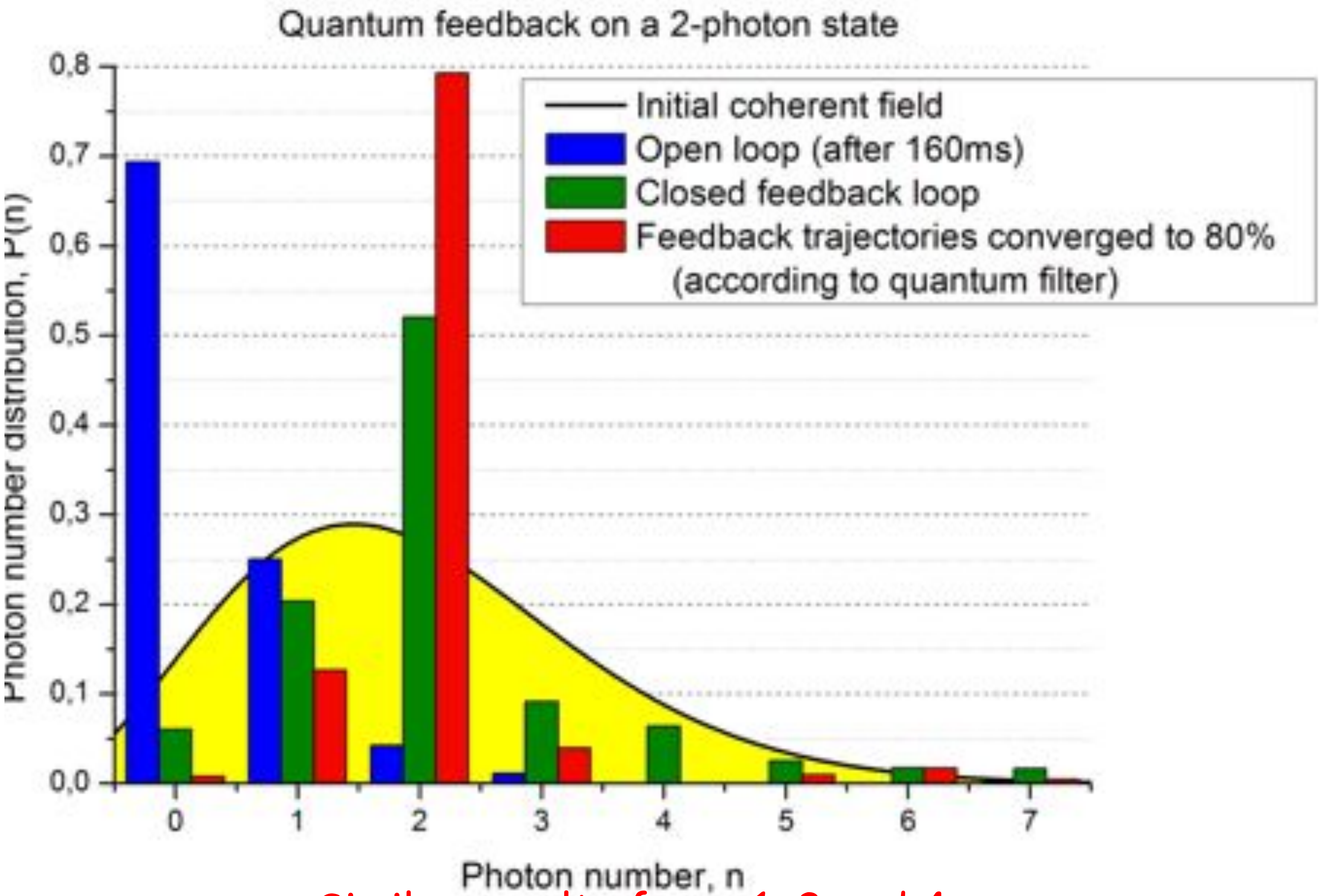


Statistical analyzis of an ensemble of trajectories



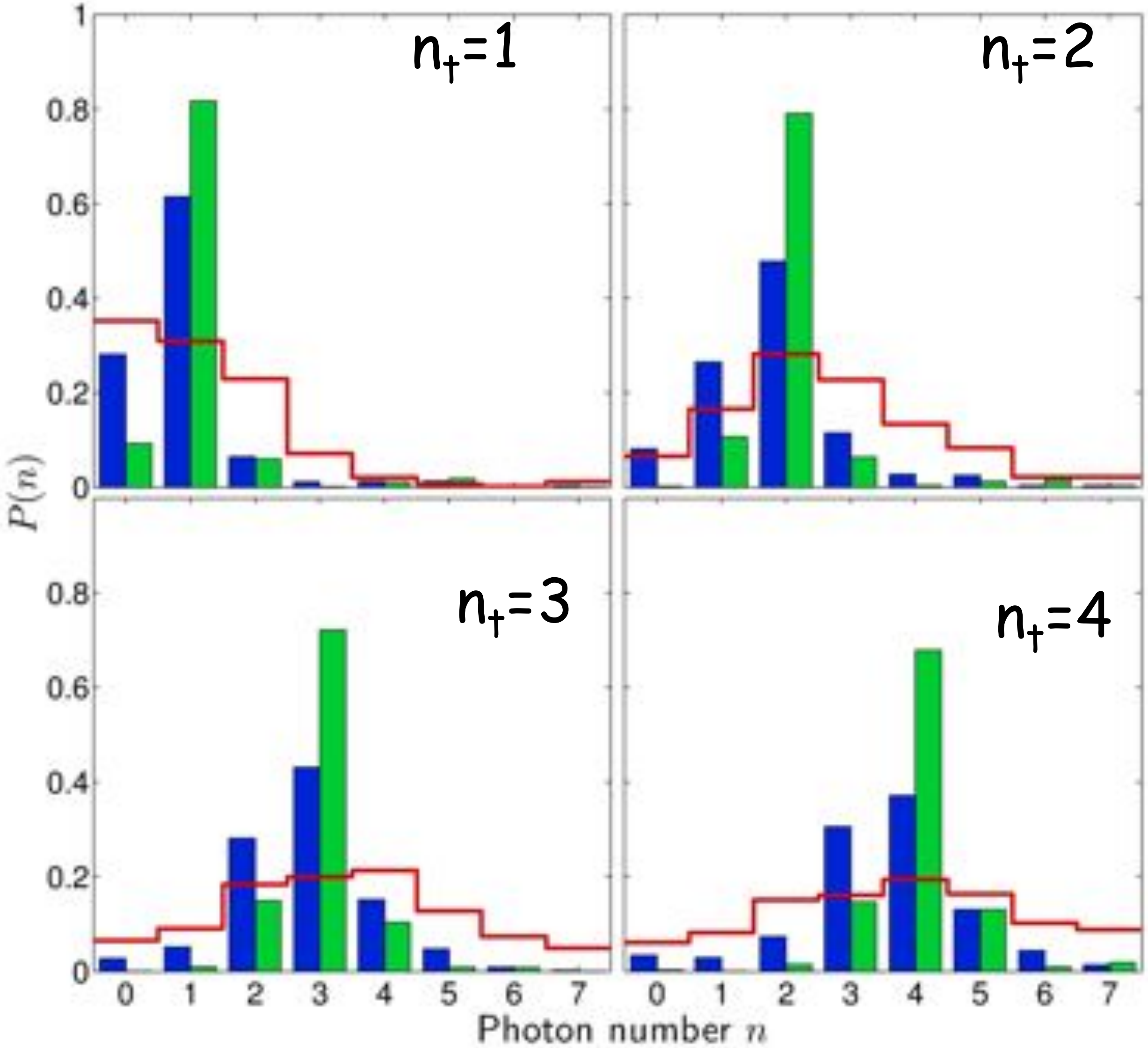

Quantum feedback on a 2-photon state
Initial coherent field
Open loop (after 160ms)
Photon number distribution, P(n)
Photon number, n
0,0
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
0
1
2
3
4
5
6
7


Quantum feedback on a 2-photon state
Initial coherent field
Open loop (after 160ms)
Closed feedback loop
Photon number distribution, P(n)
0,0
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
0
1
2
3
4
5
6
7
Photon number, n



Similar results for n=1, 3 and 4…

Photon number probability distributions
(statistical average over large number of trajectories)



Initial field in red

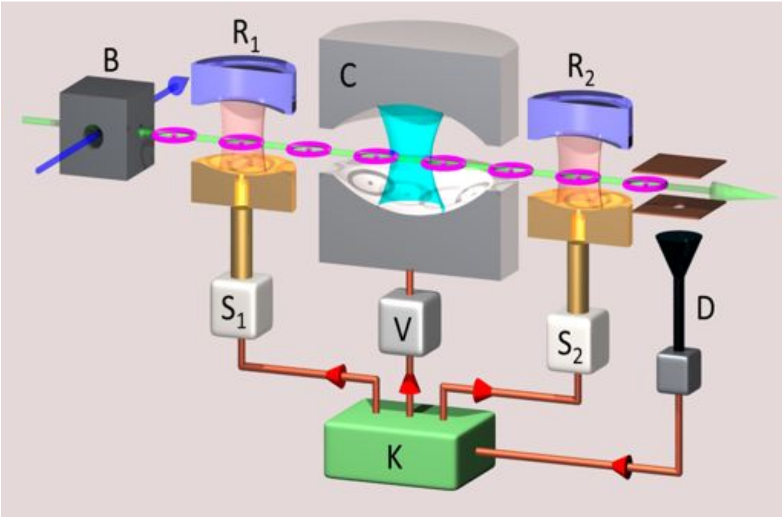
Field after controller announces convergence in green

Steady state field in blue

IV-C

Quantum feedback by atomic emission or absorption: a micromaser locked to a Fock state

The three kinds of atoms

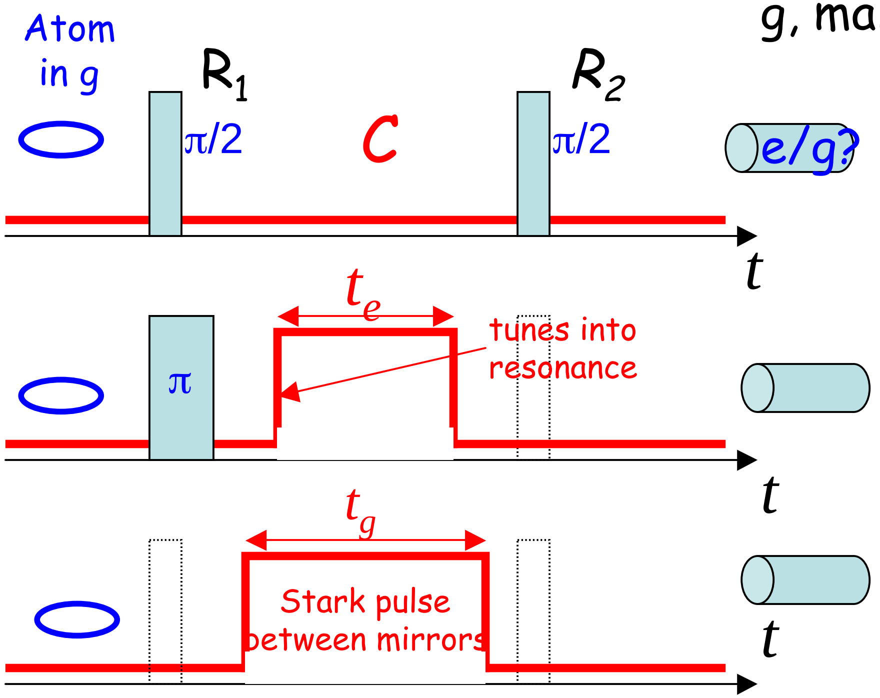


The algorithm relies on three kind of actions:

Non-resonant sensor atoms, prepared in state superposition in R_1, perform QND measurements in Ramsey interferometer

Resonant emitter atoms, prepared in state e in R_1, make the field jump up in Fock state ladder.

Resonant absorber atoms, prepared in state g, make field jump down in Fock state ladder.



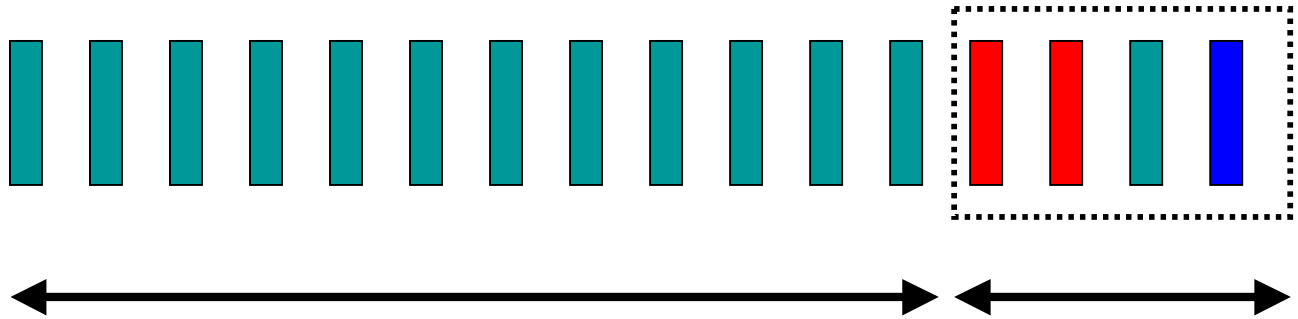
sensor

Emitter actuator

Absorber actuator

Switching between these three modes is controlled by K via microwave pulses applied in R_1,R_2 by S_1 and S_2 and dc voltage V across C mirrors (Stark tuning of atomic transition in and out of resonance)

The quantum feedback loop with atomic sensors and actuators



12 QND sensor samples
(0,1 or 2 atoms in each)

4 control samples
(K decides which
mode is best)

It requires several atoms to acquire info about photon number, but in principle only one atom to correct by ± 1 photon: hence, many more sensors than emitter/absorbers

How does K estimates the field state, computes the distance to target and decides what to do with the four control samples in each loop?

Updating the field estimation (an exercise on Bayes logic)

The field, initially in vacuum and being coupled to resonant atoms entering C with no phase information, does not build any coherence between Fock states. Its density matrix remains thus diagonal in Fock state basis and the field quantum state is entirely determined by its photon number distribution p(n). We must then only find out how p(n) is updated when a sample is detected.

1. Sensor sample: The field updating is fully determined by the characteristics of the Ramsey interferometer. Let us start by recalling the ideal conditional probability to detect an atom in j (j=0 if e, j=1 if g) provided they are n photons:

$$\pi_{S\,ideal}\left(j\,|\,n\right) = \cos^2(n\Phi_0 - \varphi_r - j\pi)/2 = \frac{1}{2}\left[1 + \cos(n\Phi_0 - \varphi_r - j\pi)\right]$$

Due to imperfections, this ideal Ramsey signal is modified by offset and finite contrast and becomes (b and c being calibrated in auxiliary experiments):

$$\pi_S\left(j\,|\,n\right) = \frac{1}{2}\left[b_r + c_r\cos(n\Phi_0 - \varphi_r - j\pi)\right] \quad ; \quad b_r \sim 1, \quad c_r < 1$$

Bayes law tells us that if the atom is found in j, then the p(n) probabity becomes:

$$p_{after}(n\,|\,j) = \pi_S\left(j\,|\,n\right)p(n)/\pi(j) \quad ; \quad \pi(j) = \sum_n\left(\pi_S\left(j\,|\,n\right)p(n)\right)$$

As already noted, p(n) is multiplied (within a normalization) by the fringe function of the interferometer. Full account is taken of imperfections by using the real (experimental) fringe function rather than the ideal one.

An exercise on Bayes logic (cntn'd)

2. Emitter: If atom is sent in e with n photons, the conditional probability to detect the atom in k (k=0 if e, k=1 if g) is (ideal Rabi oscillation):

$$\pi_{emit}\left(e \to k \mid n\right) = \cos^2\left[\left(\Omega\sqrt{n+1}\,t_e - k\pi\right)/2\right] = \frac{1}{2}\left[1+\cos\left(\Omega\sqrt{n+1}\,t_e - k\pi\right)\right] \quad ; \quad t_e : adjustable\ Rabi\ flopping\ time$$

Bayes law yields proba. that field *had* n photons conditionned to emitter found in j:

$$p_{before}\left(n \mid e \to k\right) = \pi_{emit}\left(e \to k \mid n\right) p\left(n\right) / \pi\left(e \to k\right) \quad ; \quad \pi\left(e \to k\right) = \sum_n \pi_{emit}\left(e \to k \mid n\right) p\left(n\right)$$

A priori proba of e→k transition

We know that field has +1 photon if k=1. Hence, the proba. after emitter crossed C:

$$p_{after}\left(n \mid e \to k\right) = \pi_{emit}\left(e \to k \mid n-k\right) p\left(n-k\right) / \pi\left(e \to k\right)$$

The photon distribution is multiplied by an oscillating function. Even if no photon is emitted (k=0, emitter found in e), the probability is changed: the information provided by atom detection modifies state even if no energy has been exchanged.

3. Absorber: If atom is sent in g with n photons, we get similarly:

$$\pi_{absorb}\left(g \to k \mid n\right) = \sin^2\left[\left(\Omega\sqrt{n}\,t_g - k\pi\right)/2\right]$$

$$p_{after}\left(n \mid g \to k\right) = \pi_{absorb}\left(g \to k \mid n-k+1\right) p(n-k+1) / \pi\left(g \to k\right)$$

Summary of results for an actuator (emitter or absorber) realizing j→k (j,k=0,1)

$$\pi_{actuator}\left(j \to k \mid n-k+j\right) = \frac{1}{2}\left[1+\cos\left(\Omega\sqrt{n-k+1}\,t_j - (j-k)\pi\right)\right]$$

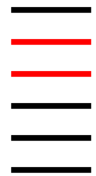
$$p_{after}\left(n \mid j \to k\right) = \pi_{actuator}\left(j \to k \mid n-k+j\right) p\left(n-k+j\right) / \pi(j \to k)$$

Imperfections alter contrast of Rabi flopping: formulas modified (calibration by preliminary auxiliary experiments)

An exemple of Bayes logic at work

Initial field density operator mixture of 3 and 4 Fock states:

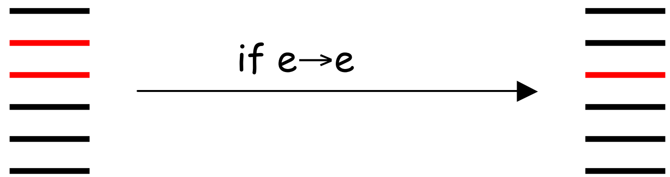
$$P(n) = \frac{1}{2}\left[\delta_{n,3} + \delta_{n,4}\right]$$



Actuator in emitting mode (prepared in e) with $\Omega\sqrt{5}\, t_e = \pi$ tuned for a π–Rabi pulse for n=4 photons.

What is the new photon number distribution if atom is detected in e? A naive approach would indicate that the field has not changed, since the atom has not. Applying the rule of last page show however that the new distribution is:

$$P_{after}(n) = \delta_{n,3}$$



Classical Baysian argument: If there were 4 photons in initial field, the π–Rabi pulse (tuned for n=4) would with certainty lead the atom from e to g and the probability to find the atom in e would be 0. The fact that we have detected atom in e eliminates this possibility and leaves with certainty the atom in the n=3 state (for which the π-Rabi pulse condition is not fulfilled). Acquisition of information about an event modifies the distribution of the causes of this event, as already shown in various examples in this course.

Computing distance to target and deciding best action

Intuitive definition of the distance to target state$|n_t\rangle$ (a functional of p(n)):

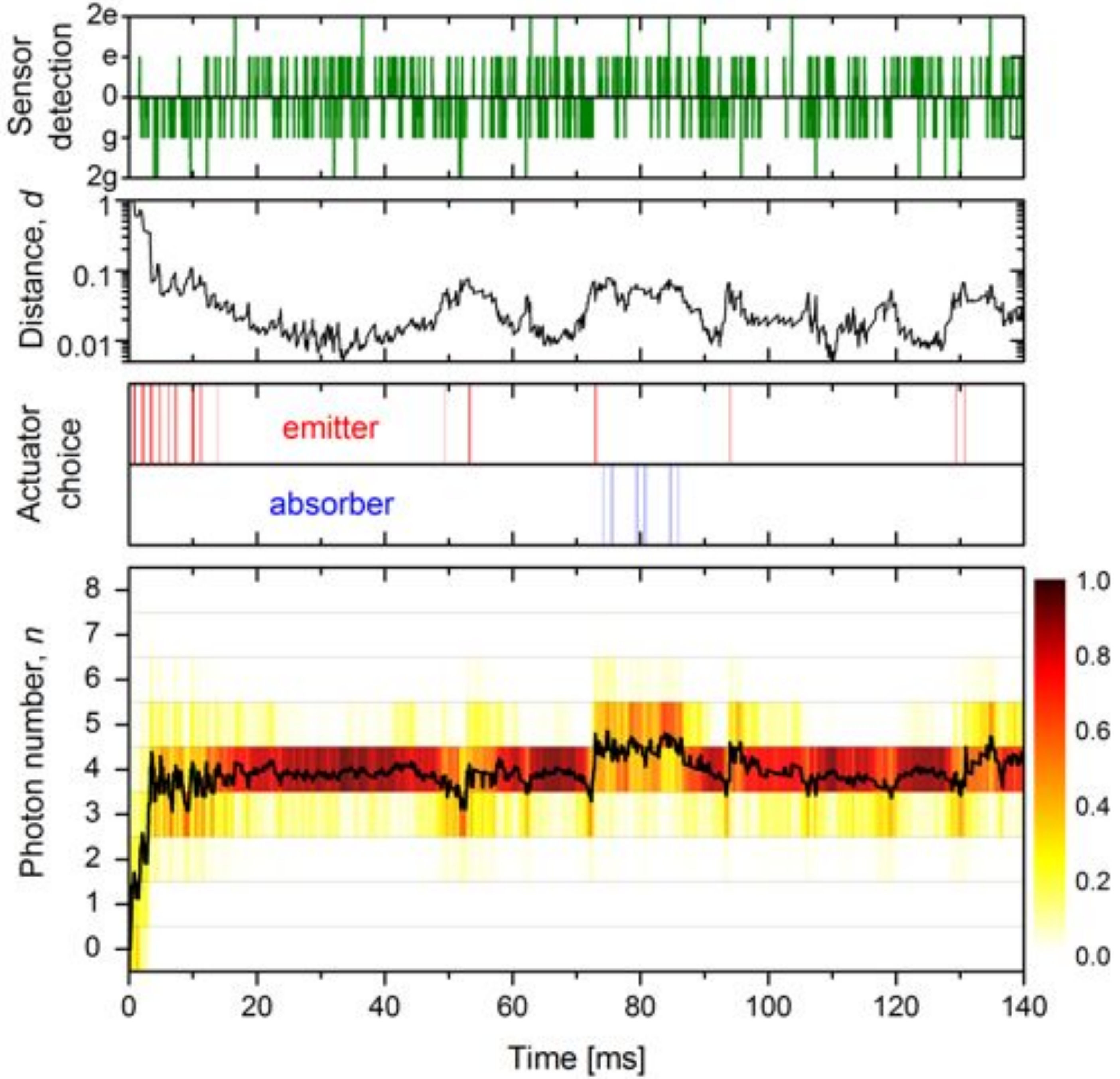
$$d\left[p(n),n_t\right]=\sum_n p(n)\left(n-n_t\right)^2=\left(\bar{n}-n_t\right)^2+\Delta n^2$$

The distance is the sum of the photon number variance and the squared difference between the mean photon number and the target photon number. The distance cancells iff the mean photon number is equal to n_t *and* the photon number variance is zero. Bringing d to zero thus realizes a quantum feedback: not only the mean field energy is driven to the target, but its fluctuation is squeezed to a strongly subpoisson regime.

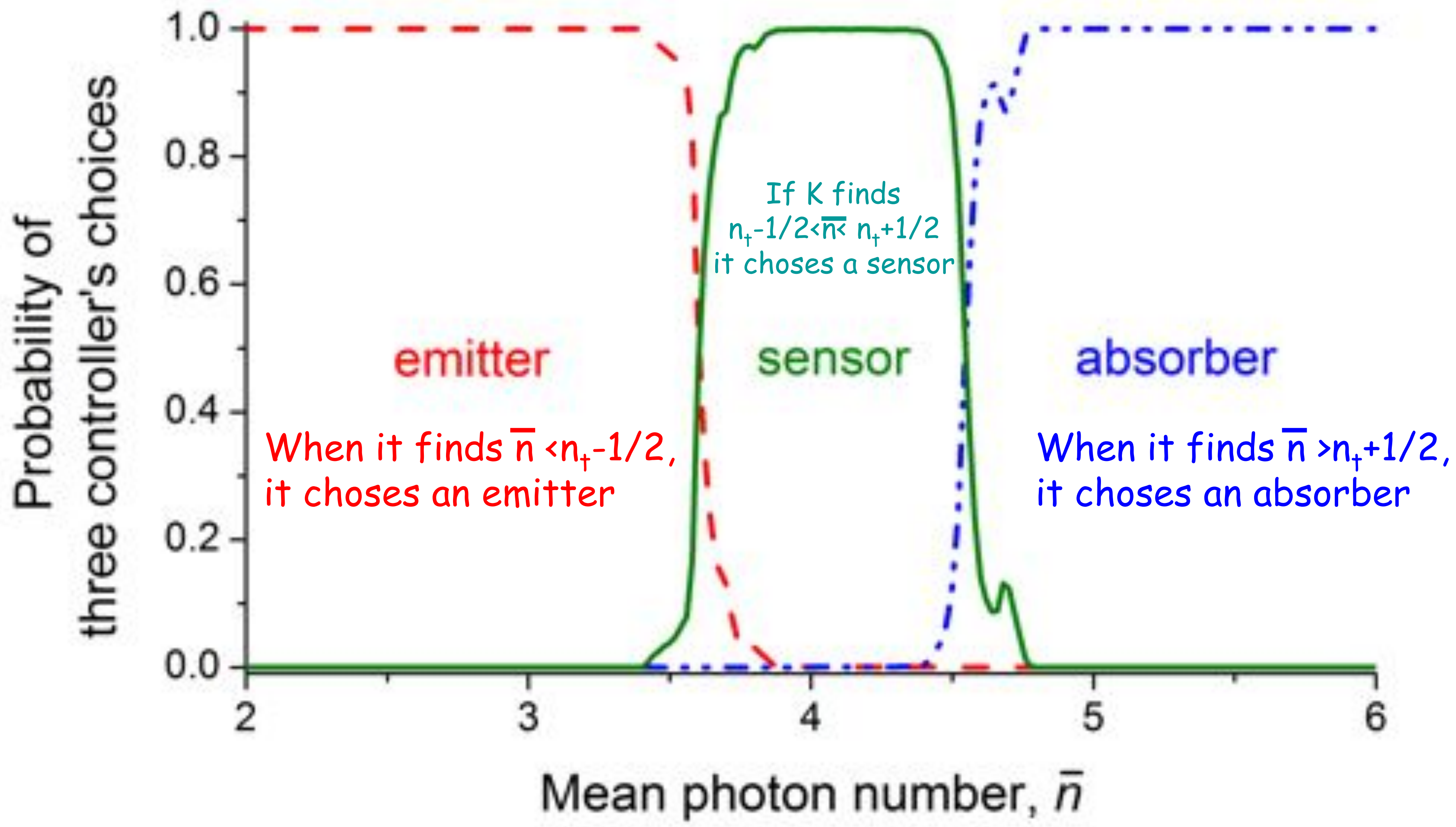
Decision algorithm: in order to chose which kind of atom to send across C for each control samples, K computes what would happen to d, on average, for an emitter or for an absorber atom (a sensor atom would not change d on average). It then choses the solution which most decreases d. If both emitter or absorber atom would increase d, it sends a sensor atom to acquire more info. on photon number. Calling $p_p(n|j)$ the photon number probability expected on average after sending atom in j and $d_p(j)$ the average expected distance, K performs the calculations:

$$\boxed{\begin{array}{c}\bar{p}_p\left(n\,|\,j\right)=\sum_k \pi\left(j\to k\right)p_{after}\left(n\,|\,j\to k\right)=\sum_k \pi_{actuator}\left(j\to k\,|\,n-k+j\right)p(n-k+j)\quad\to\quad \bar{d}_p=\sum_n (n-n_t)^2\,\bar{p}_p\left(n\,|\,j\right)\\ d_p(j)<d\to K\text{ choses actuator atom in } j\,;\,d_p(j)\geq d\ (j=0\text{ and }1)\to K\text{ choses sensor atom}\end{array}}$$

Locking the field to the n=4 Fock state

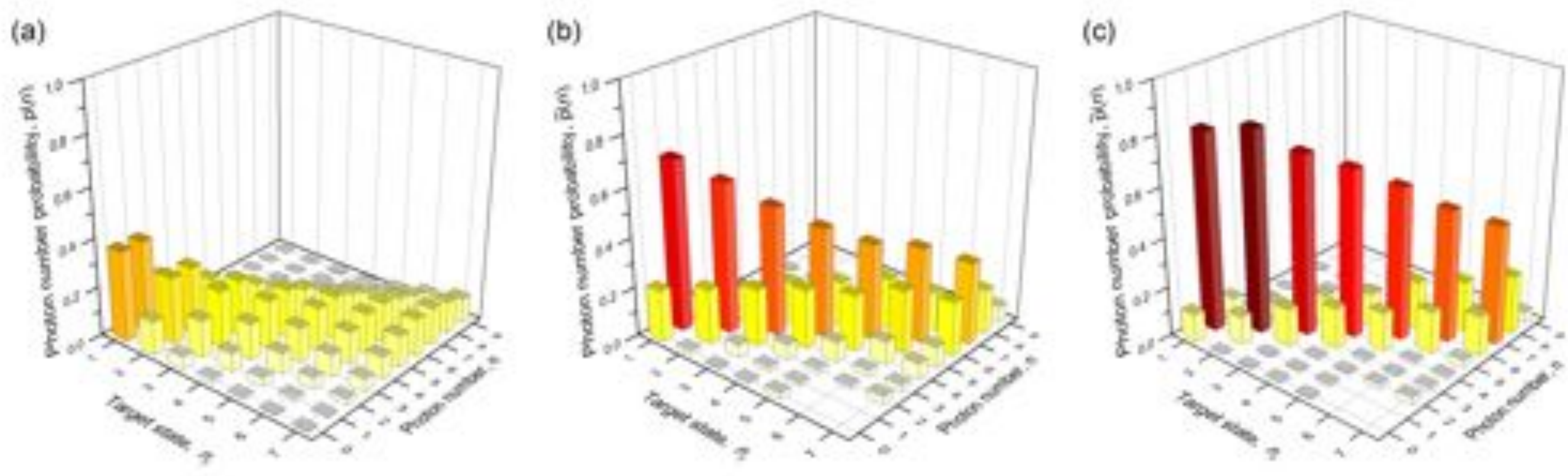


K follows an intuitive procedure…



System behaves as a micromaser with an adjustable ratio of emitter and absorber atoms controlled to lock field to Fock state

Statistical analysis of 4000 trajectories for each target state

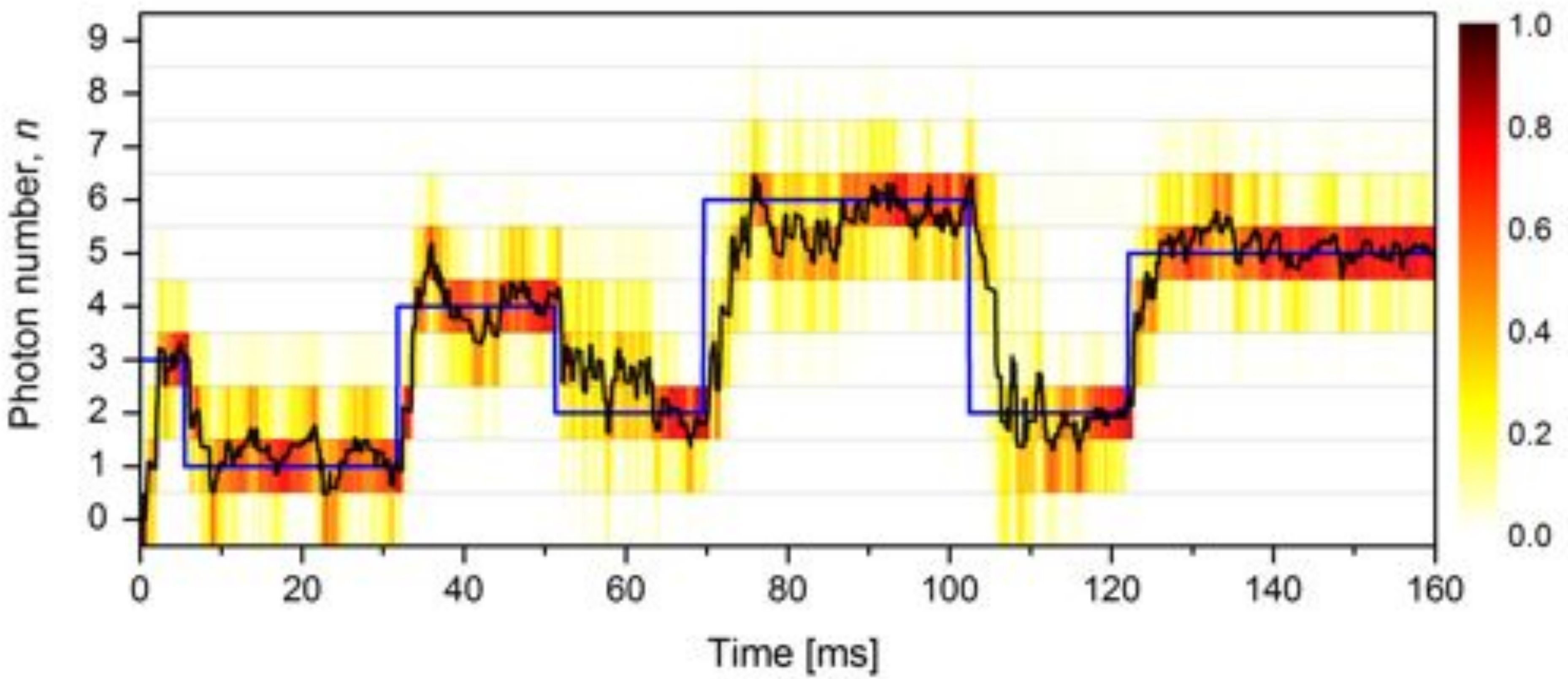


For comparison, Poisson distributions with mean photon numbers 1 to 7

Photon number distributions for the targets n_t=1,2,3,4,5,6,7 when quantum feedback is stopped at fixed time

Photon number distributions for same targets when quantum feedback is interrupted after K announces successful locking (with fidelity >0.8)

Programming a walk between Fock state by changing the target state (here the sequence n= 3,1,4,2,6,2,5)



Conclusion of 4th lecture

In this lecture, I have demonstrated how quantum feedbak can be implemented in Cavity QED to prepare and stabilize Fock states against quantum jumps. Two methods have been tried.

In the first, the actuator is a classical source injecting small pulses of coherent radiation in the cavity. The corrections of $\Delta n = \pm 1$ quantum jumps are achieved by incremental steps made of pulses with positive or negative amplitudes, many pulses of decreasing intensity being required to make the field converge back into a Fock state. The transient off-diagonal density matrix elements generated in the process are destroyed by the quantum collapses induced by the dispersive probe atoms. The process takes a few tens of milliseconds, making the procedure relatively slow and impossible to implement for $n>4$.

In the second method, the actuators are single resonant atoms able to inject or substract a photon in one step, making the procedure more reactive and faster. Fock states up to $n=7$ have been prepared and protected in this way.

Extending the method to protect other kinds of states, such as Schrödinger cat states is an interesting field of investigation.

References for this lecture:

Theory of quantum feedback in CQED: I.Dotsenko et al, Phys.Rev.A 80, 013805 (2009)

Experiment with classical actuators: C.Sayrin et al, Nature, 477, 73 (2011)

Experiment with quantum actuators: Xingxing Zhou et al, to be published (2012)